



# **Australian Vertical Working Surface (AVWS)**

## **Technical Implementation Plan**

Version 1.0

Intergovernmental Committee on Surveying and Mapping (ICSM)

Permanent Committee on Geodesy (PCG)

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## Document History

Version	Written by	Date
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# 1. Purpose

The purpose of this document is to provide a publicly accessible resource of:

- information explaining the need for the Australian Vertical Working Surface (AVWS);
- definition of AVWS and its relationship to the Australian Height Datum (AHD); and
- information, tools, products and services to enable people to access AVWS.

The document addresses many of the complex, geodetic and technical issues associated with the implementation of a vertical reference surface and is therefore intended for those with expertise in geodesy or the geospatial industry.

# 2. Motivation for introducing AVWS

The Australian Government has committed \$225m to Geoscience Australia to implement the Positioning Australia program to provide accurate and reliable positioning to everyone.

In anticipation for the growing use and reliance on positioning technology, the Permanent Committee on Geodesy is leading the upgrade of a number of elements of Australia's Geospatial Reference System including the introduction of AVWS. The AVWS is a model that provides the height difference between the ellipsoid and the gravimetric geoid. It differs from AUSGeoid2020, which provides the offset between the ellipsoid and Australian Height Datum (AHD), by between -1 to 1 m throughout Australia.

The AVWS is not replacing AHD, but instead is an alternative reference for heights for those who wish to use it. A recent user requirements study (Brown et al., 2019a; Brown et al., 2019b) found that AHD is not capable of meeting some user requirements; predominantly when working over distances greater than 10 km. This is predominantly due to localised errors and distortions in the AHD. When deriving AHD heights from GNSS and AUSGeoid, users are able to achieve accuracy of 6-13 cm. The alternative, AVWS, is accurate to 4-8 cm and will improve over time as data is added (predominantly from airborne gravity).

# 3. Height Fundamentals

Height determination in Australia requires a level of care due to the number and types of datums to which heights can be referred, including:

**Ellipsoid:** Simplified mathematical representation of the Earth often used as a reference surface for positioning, navigation, map projections and geodetic calculations. Ellipsoidal heights  $h$  are the distance between the ellipsoid and point of interest measured along a straight line perpendicular to the ellipsoid.

**Geoid:** Surface of equal gravitational potential (or equipotential) that closely approximates mean sea level. Heights with respect to the geoid are known as orthometric heights  $H$  and are the curved line distance between the geoid and point of interest measured along the plumbline.

**Quasigeoid:** Non-equipotential surface of the Earth's gravity field closely aligned to the geoid with differences up to about 3.4 m in the Himalayas (Rapp, 1997) and 0.15 m in Australia (Featherstone and Kirby, 1998). Heights with respect to the quasigeoid are known as normal heights  $H^*$  and are the curved line distance between the quasigeoid and point of interest measured along the plumbline.

**Mean Sea Level:** Mean Sea Level (MSL) is an observed tidal datum and is used as the conventional reference surface to which heights on the terrain (e.g. contours, heights of mountains, flood plains, etc.) and other tidal datums are related.

**Mean Sea Surface:** Mean Sea Surface (MSS) is the sum of the geoid (closely approximated by MSL) and Mean Dynamic Topography (MDT) which describes the thermodynamic motion of the oceans.

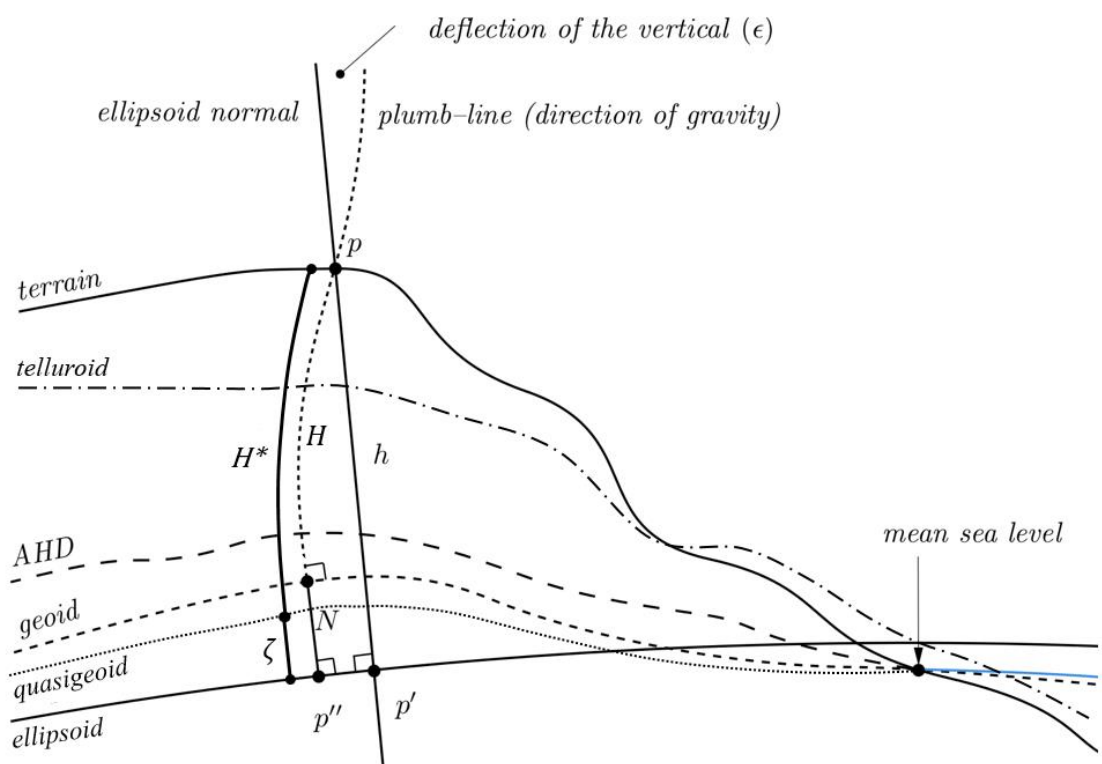


Figure 1: Heights can be observed or derived with respect to an ellipsoid, geoid or quasigeoid surface.

### 3.1 Physical Height Datums

Fluid will flow according to gravity potential making the geoid (a surface with equal gravity potential at every point) a useful datum for heights. An ellipsoid does not have equal gravity potential. In fact, across Australia, the difference between the geoid and the ellipsoid is between -30 and +80 m (Figure 2). For this reason, ellipsoidal heights observed using Global Navigation Satellite Systems (GNSS) often need to be converted to physical heights (a height with respect to the Earth's gravity potential) using a model of the geoid or quasigeoid.

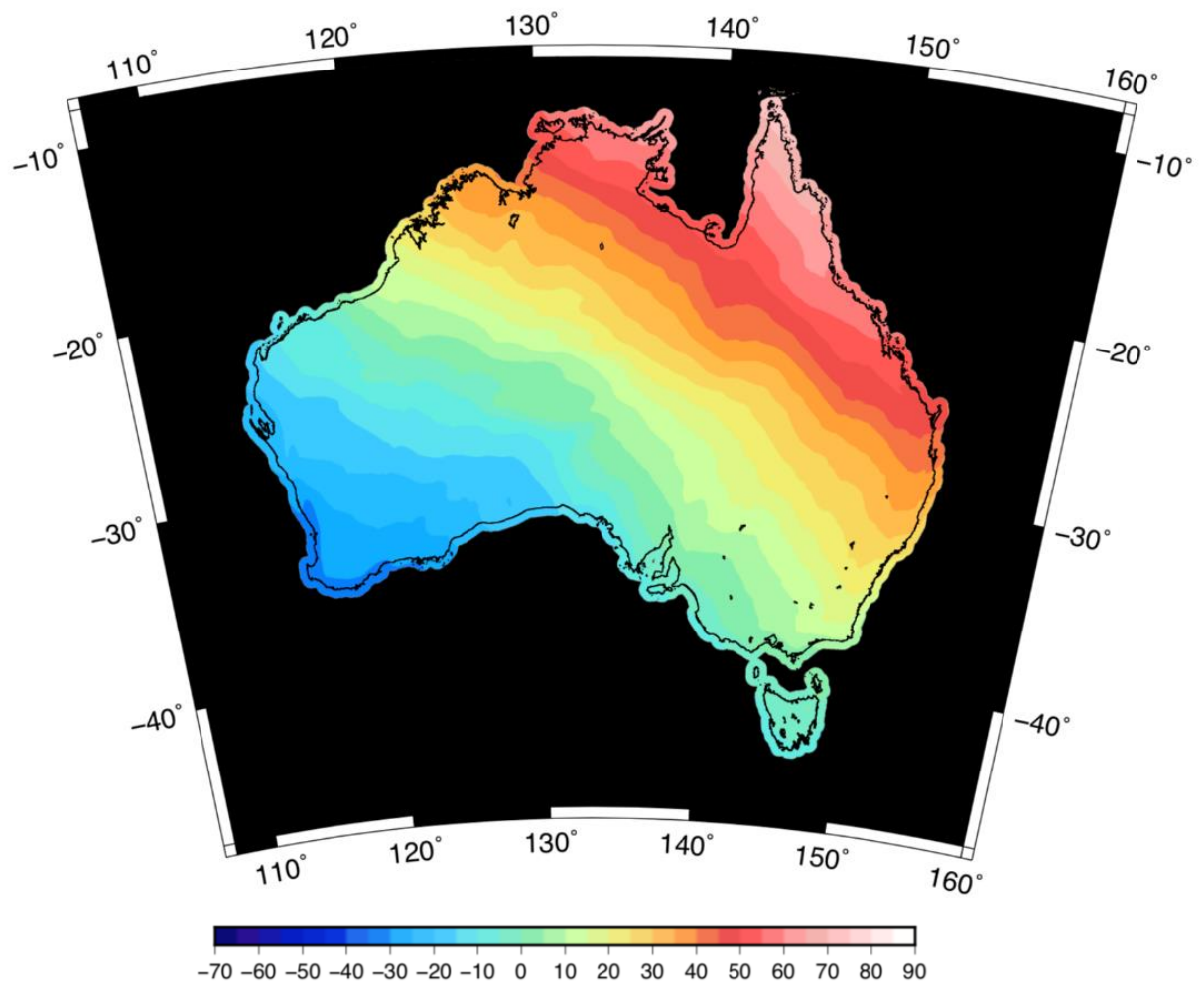


Figure 2: An ellipsoid does not have equal gravity potential. In fact, across Australia, the difference between the geoid and the ellipsoid is between -30 and +80 m.

### 3.2 Height Systems and Height Datums

A height system is a coordinate system used to define the height of a point above or below a reference surface. Its definition varies according to the reference surface chosen (e.g. geoid) the path along which the height is measured (e.g. plumbline). A height datum is the practical realisation of a height system (e.g. Australian Height Datum).

A height system could have many realisations (datums) as new theories, computational process and data become available. Generally, each new height datum is a better (more accurate, reliable, robust and fit for purpose) realisation of the height system. Although there is only one national height datum, AHD, there are many other height datums used in Australia (mining, rail, road authorities, marine etc.). It is therefore important to clearly define the following elements of a height datum:

- the height system, including a reference ellipsoid and theoretically true equipotential surface (e.g.  $W_0 = 62,636,856 \text{ m}^2 \text{ s}^{-2}$ ); and
- the information used in an attempt to physically realise the height system. In the case of AHD, this information includes:
  - Mean Sea Level (MSL) observations at 32 tide gauges around Australia; and

- Over 200,000 km of levelling used to transfer MSL heights throughout Australia.

### 3.3 Gravity Potential

The gravitational potential energy at a location is equal to the work (energy transferred) per unit mass needed to move an object from one point to another point.

The geopotential number  $C$  is the basis of all height systems in physical geodesy. A geopotential number is the difference in gravitational potential energy between a point  $P$  (e.g. on the Earth's surface)  $W_p$  and potential on the reference surface  $W_0$  (e.g. the geoid),

$$C = W_p - W_0$$

The negative of the geopotential number ( $m^2/s^2$ ), divided by some value of gravity ( $m/s^2$ ) yields a unit of length ( $m$ ).

## 4. Geoid

There are an infinite number of surfaces of equal gravity potential radiating out from the centre of mass of the Earth to outer space. The geoid is the surface of equal gravity potential which is the best fit to mean sea level and is denoted by  $W_0$  (units  $m^2s^{-2}$ ) (Figure 3).

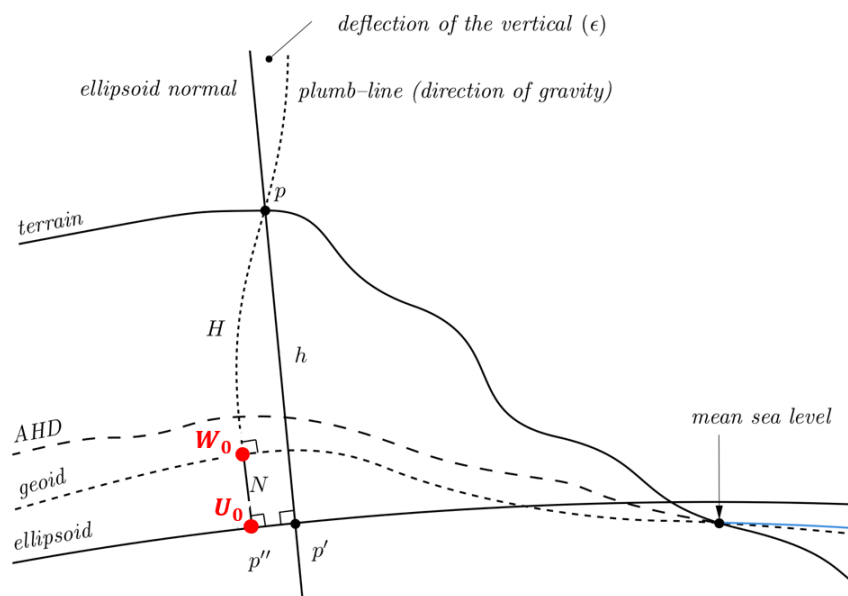


Figure 3: The geoid is the surface of equal gravity potential which is the best fit to mean sea level and is denoted by  $W_0$ .

Heights with respect to the geoid are called orthometric heights  $H$ . To approximately compute physical heights from GNSS, the geometric distance between the ellipsoid and the geoid is known as the geoid undulation  $N$  needs to be subtracted from the ellipsoidal height  $h$  (Figure 4).

$$H \approx h - N$$

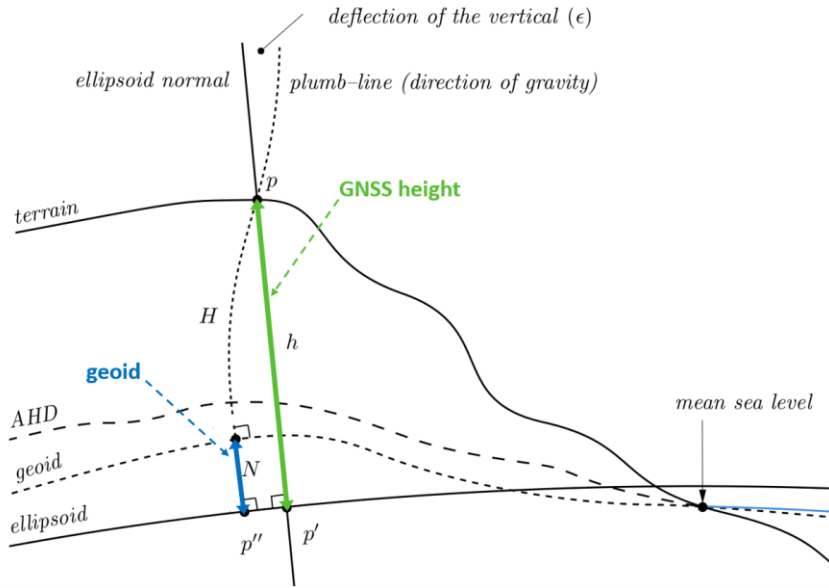


Figure 4: The geometric distance between the ellipsoid and the geoid is the geoid undulation.

There are a wide range of geoid models which have been developed to enable the conversion of geometric ellipsoidal heights to physical heights including global gravity models such as the Earth Geopotential Model 2008 (EGM2008). EGM2008 has an absolute accuracy of about 20 cm. (Yi and Rummel, 2013). In cases where a more accurate datum for physical heights is required, countries have developed national or local geoid models which use a global gravity model, and augment it with local data such as terrestrial and airborne gravity data.

#### 4.1 Developing a geoid model

The disturbing potential,  $T$  is the difference between the Earth's gravity potential field  $W$  and the gravitational potential field of the ellipsoid  $U$ .

$$T = W - U$$

When  $T$  is known on the surface of the geoid, the geometric separation / geoid undulation ( $N$ ) between the geoid surface and the ellipsoid is given by;

$$N = \frac{T}{\gamma}$$

where  $\gamma$  is the *normal gravity* (i.e. the gradient of the ellipsoidal potential) evaluated on the surface of the ellipsoid.

The potential  $W$ , and therefore the disturbing potential  $T$ , cannot be measured directly. But the gradient of the potential,  $\frac{dW}{dr}$  (i.e. the familiar gravity value  $\approx 9.8 \text{ ms}^{-2}$ ) can be measured using gravimeters.

We define the *gravity anomaly*  $\Delta g$  as the difference between measure gravity on the geoid surface and normal gravity  $\gamma$  on the ellipsoid surface (Figure 5).

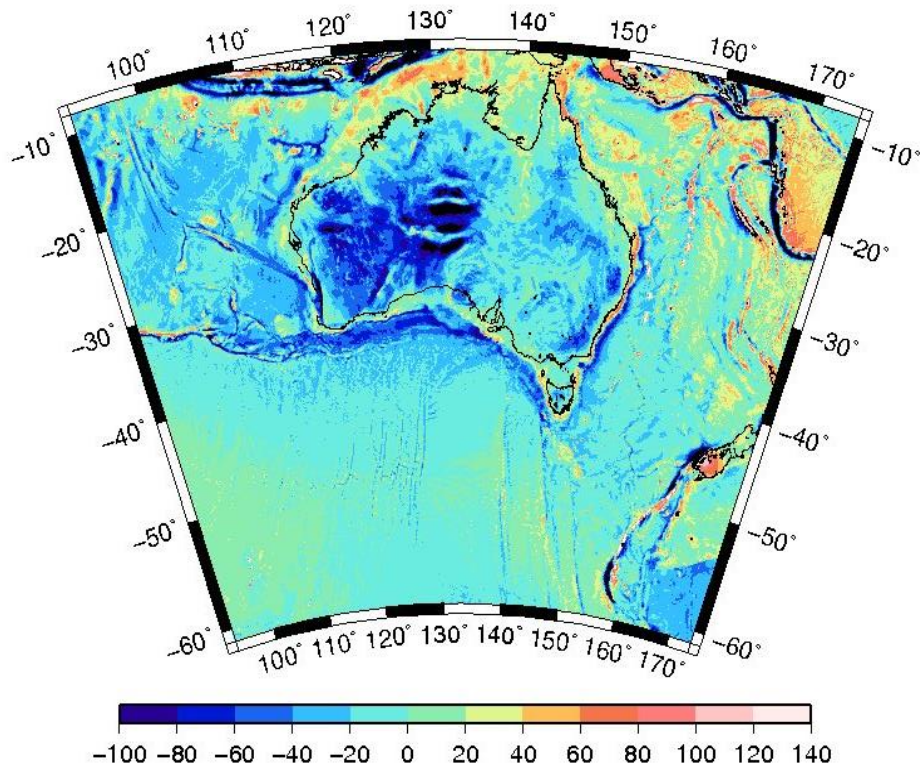


Figure 5 - Gravity anomalies over the Australian continent.

When the gravity anomalies are known on the geoid over the surface of the whole Earth, there is a mathematical relationship between them and the disturbing potential. This is known as Stokes integral (Moritz, 1980).

$$T = \kappa \int_{\sigma} \Delta g S(\psi) d\sigma$$

In practice, only long wavelengths of  $\Delta g$  are available over the whole Earth. This means only long wavelength models of the disturbing potential can be determined globally. High resolution geoid models are developed locally via the remove compute restore technique, where higher resolution gravity data are available. i.e.

$$T = \kappa \int_{\hat{\sigma}} (\Delta g - \Delta g_{Long}) S(\hat{\psi}) d\sigma + N_{Long}$$

Where  $\Delta g_{Long}$  and  $N_{Long}$  are gravity anomalies and geoid undulations from a long wavelength global model,  $S(\hat{\psi})$  is a modified form of  $S(\psi)$  where long wavelengths have been removed,  $\hat{\sigma}$  is the local region the higher resolution gravity data are available.

## 4.2 Orthometric Height System

The orthometric height system is compatible with a geoid model. An orthometric height  $H$  is the curved line distance between the geoid and point of interest measured along the plumbline and computed by,

$$H = C/\bar{g}$$

where the geopotential number  $C$  is divided by the integral mean of gravity taken along the plumbline  $\bar{g}$ .

NOTE 1: In the case of an orthometric height system, computation of the geopotential number requires gravity observations.

NOTE 2: Given that orthometric heights require information of the Earth's gravity acceleration along the length of the plumbline through the topography, it is impossible to realise in practice.

NOTE 3: Helmert orthometric height systems use an approximation of the Earth's gravity field and are not truly orthometric height systems.

## 5. Quasigeoid

Recognising that evaluating  $W_p$  on the geoid is practically impossible to do, Molodensky (1945) introduced an alternative theoretical surface called the quasigeoid. For the determination of the quasigeoid all the computations are done, not on the geoid surface but, on the surface of the Earth. Molodensky's approach deals only with the external field and needs only to know the geometry of the external field. The normal gravity is evaluated on the surface of the telluroid.

### **Def: Telluroid**

- *The telluroid is a theoretical surface:*
  - *where the normal potential gravity is equal to the true gravity potential on the Earth's surface i.e.  $U_{p_3} = W_{p_4}$  and on the same plumb line; and*
  - *looks like the Earth surface except that it is displaced from the Earth surface by the quasigeoidal height (Figure 6).*

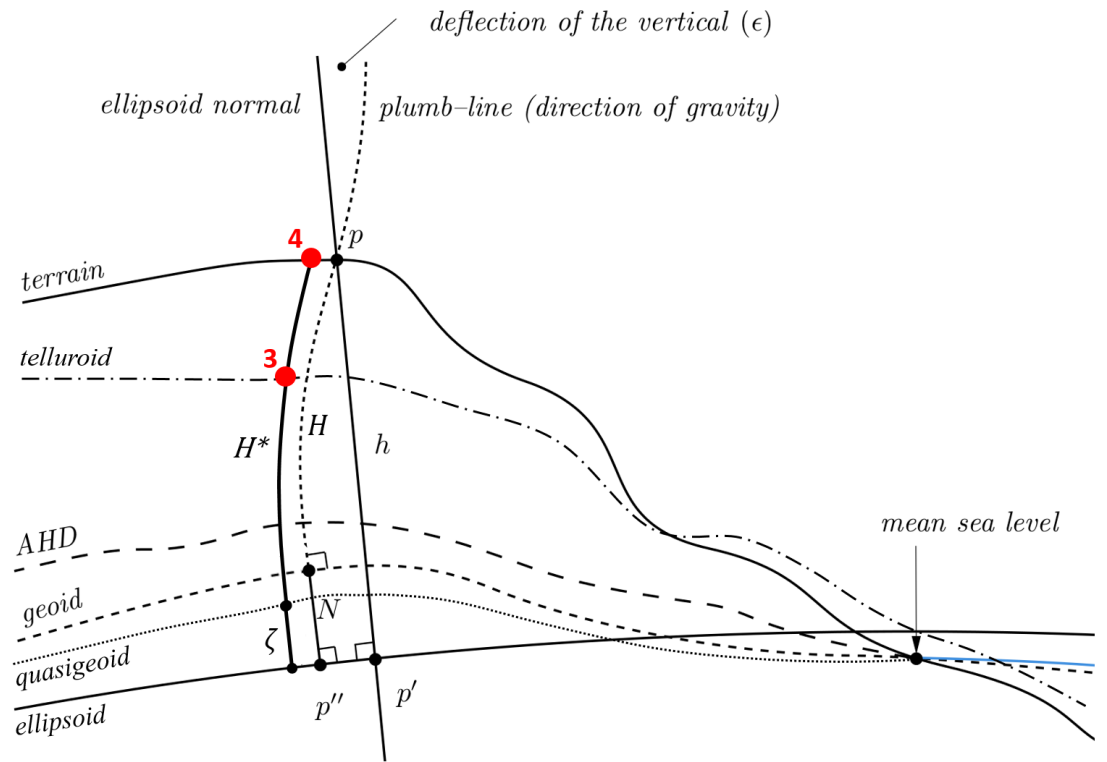


Figure 6: The telluroid is a theoretical surface where the normal potential gravity is equal to the gravity potential of the Earth on the Earth's surface i.e.  $U_{p_3} = W_{p_4}$ .

Offshore, where there is no topography, the quasigeoid agrees with the geoid. The quasigeoid can, in theory, be determined exactly (i.e. without any approximations). It provides the reference surface for normal heights  $H^*$  which can be determined from levelling and gravity observations, or derived normal heights from GNSS and a quasigeoid model. Onshore, it differs from the geoid by 1-2 cm in flat terrain up to 10 cm in steep topography (Figure 7).

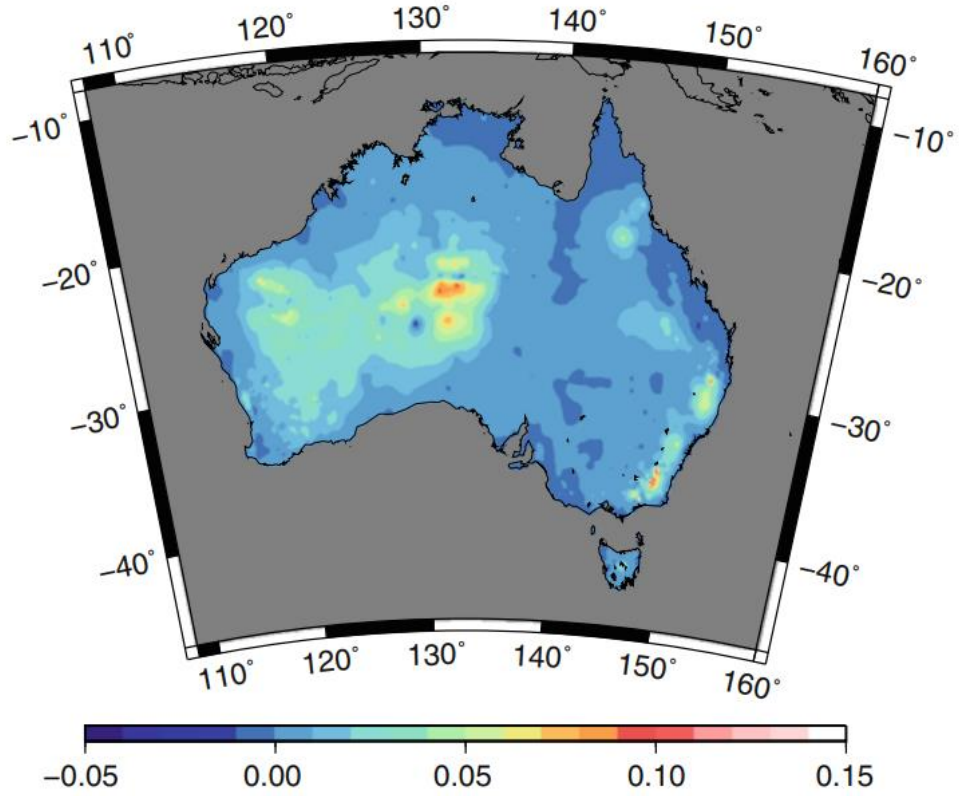


Figure 7: Differences between Helmert Orthometric (from geoid) and Normal Heights (from quasigeoid) (in m) over Australia from Filmer et al. (2010).

To compute normal heights from GNSS, the geometric distance between the ellipsoid and the quasigeoid is known as the height anomaly  $\zeta$  needs to be subtracted from the ellipsoidal height  $h$ .

$$H^* = h - \zeta$$

In the same way that a geoid model gives geoid undulation  $N$  at any point, quasigeoid models gives height anomalies  $\zeta$  at any point. The normal height of a point on the topographical surface is defined as the height of the corresponding point on the telluroid above the reference ellipsoid, measured along the normal plumbline. However, normal heights may equivalently be seen as heights of the topographical surface above the quasigeoid, also measured along the normal plumbline.

## 5.1 Developing a quasigeoid model

On the Earth's surface the disturbing potential is given by,

$$T_{p_4} = W_{p_4} - U_{p_3} + \zeta\gamma$$

and so

$$\zeta = \frac{T_{p_4}}{\gamma}.$$

Here,  $\gamma$  is the normal gravity, evaluated on the telluroid.

## 5.2 Normal Height System

The normal height system was proposed in 1954 by Molodensky et al. (1962) to overcome the problem in orthometric heights of having to determine the mean value of gravity along the plumbline. The normal height  $H^*$  is the distance between the quasigeoid and the point of interest measured along the curved normal and computed by,

$$H^* = C / \bar{\gamma}$$

where the geopotential number  $C$  is divided by average normal gravity  $\bar{\gamma}$  along the plumbline.

## 5.3 Normal-Orthometric Height System

The normal-orthometric height  $H^{NO}$  is distance between the quasigeoid and the point of interest measured along the curved normal gravity  $\gamma$  plumbline and computed by,

$$H^{NO} = C_\gamma / \bar{\gamma}$$

In contrast to orthometric and normal height systems, which require gravity observations to be taken along the levelling traverse in order to derive the geopotential numbers (or normal or orthometric corrections), geopotential numbers,  $C$ , are replaced by differences in normal potential  $C_\gamma$  (known as normal-geopotential or spheropotential numbers) and gravity is replaced by normal gravity (integral mean value of normal gravity taken along the normal plumbline between the quasigeoid and point of interest) (Featherstone and Kuhn, 2006).

The difference between normal heights and normal-orthometric heights is due to the gravity correction applied to levelling data. Normal heights require a location specific gravity value, whereas, normal-orthometric heights are derived using a gravity value based on the normal gravity field (Rapp, 1961). The difference between these two height systems is shown in Figure 8.

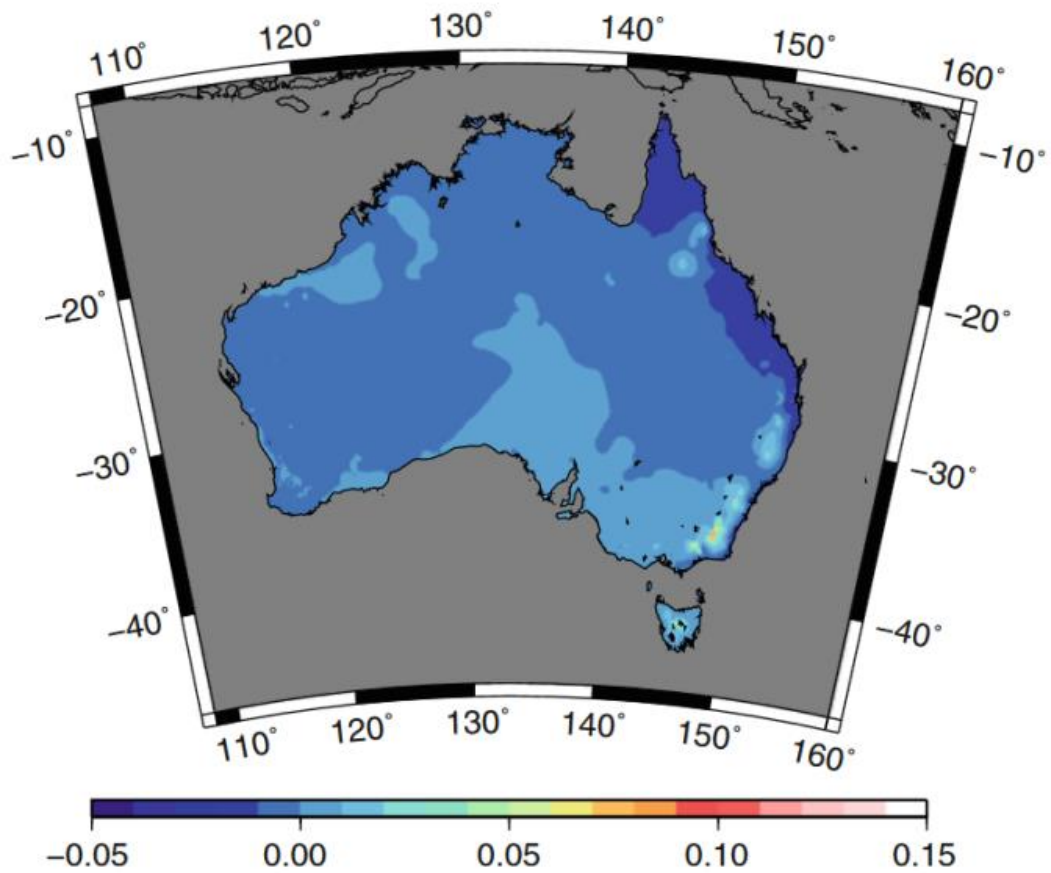


Figure 8: The difference between normal and normal-orthometric heights over Australia (from Filmer et al, 2010) in metres. Stats: [min: -2.4 cm; max: 17.7 cm; std: 1.2 cm].

## 6. Australian Vertical Working Surface

### 6.1 AVWS Purpose

The purpose of AVWS is to provide a reference surface for heights which:

- works seamlessly onshore and offshore;
- is directly compatible with Global Navigation Satellite Systems (GNSS);
- is continuously improved over time; and
- is more accurate because it does not suffer from biases and distortions in the Australian Height Datum (AHD).

### 6.2 AVWS Definition

The AVWS is model of the quasigeoid. The AVWS model  $\zeta_{AVWS}$  can be used to transform ellipsoidal heights  $h$  (from GNSS observations) to AVWS heights  $H_{AVWS}^*$ .

$$H_{AVWS}^* = h - \zeta_{AVWS}$$

The AVWS is defined on a 1 arc minute grid from 8°(S) to 61°(S) and 93°(E) to 174°(E). It has been determined from (approx. 1.8 million) gravity values provided in the Australian National Gravity Database onshore, Sandwell et al. (2014) satellite altimetry derived gravity anomaly values offshore, global gravity model (EGM2008), and the national digital elevation model DEMH1s. A detailed description of the procedure used to create the model is given in Featherstone et al. (2018).

### 6.3 Issues with AHD

The Australian Height Datum (AHD) is known to have a number of biases and distortions which mean GNSS users are only capable of deriving AHD heights with accuracy of 6-13 cm across Australia. These biases and distortions are attributable to:

- The ocean's time-mean dynamic topography (MDT).
- Short tide gauge observation periods.
- The zero reference of the AHD (MSL at 32 tide gauges) is not coincident with an equipotential surface (e.g. the geoid). This largely manifest in a north-south tilt of ~0.7 m in the AHD relative to the geoid across the continent.
- Local and regional distortions due to systematic and gross errors in the Australian National Levelling Network (ANLN) that propagated through the national network adjustments.

These non-gravimetric artefacts are inconsistent over large distances (e.g. greater than 10 km) and means that GNSS users are only capable of deriving AHD heights with accuracy of 6-13 cm across Australia.

Uncertainty in the national height datum of this magnitude makes AHD inappropriate for some applications that require a more accurate reference surface. In response to this Geoscience Australia led a user requirements study with FrontierSI to investigate current and future requirements for physical height determination and transfer in Australia (Brown et al. 2019a; Brown et al. 2019b; McCubbine et al. 2019).

In addition to the aforementioned deficiencies, feedback from the survey included commentary on the lack of levelling benchmarks. In some regions, physical monuments have never been established or have been destroyed. In these areas levelling users are unable to tie into the datum easily, and for GNSS users the geometric component of the AUSGeoid2020 model is not adequate. Furthermore, users commented on difficulties combining data in the littoral zone. AHD is only an onshore datum. This is problematic for datasets which cover on and offshore regions (e.g. bathymetric and topographic elevation models).

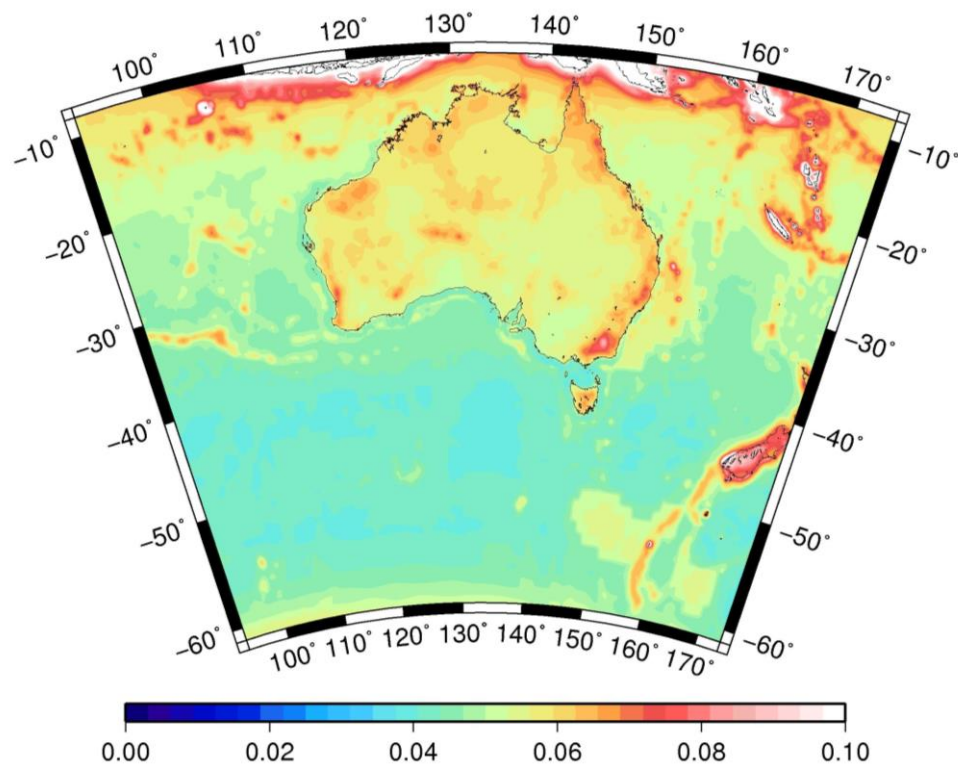
Overall, the results of the study indicated that AHD is still fit for purpose for tasks over short distances (less than about 10 km) for projects such as cadastral, civil engineering, construction and mining while users are less satisfied when working over larger areas (greater than about 10 km) for environmental studies (e.g. flood, storm modelling), LiDAR surveys, geodesy, hydrography.

## 6.4 Benefits of AVWS

In comparison to AHD, AVWS is:

- Internally consistent, being defined solely from gravity field measurements i.e. it is not contaminated with non-gravimetric artefacts due to mean dynamic topography and local distortions in levelling networks.
- Not reliant upon benchmark heights.
- Defined seamlessly on and offshore.

For these reasons it better meets the needs identified during the user requirements survey to establish or transfer accurate heights over long (>10 km) distances. Additionally, the AVWS model is provided with a corresponding map of uncertainty values formally propagated from the raw data sources through each stage of the computation (Featherstone et al., 2018). The uncertainty in the AVWS model is 4-8 cm across mainland Australia. AUSGeoid2020 on the other hand has uncertainty of 6-13 cm (Figure 9).



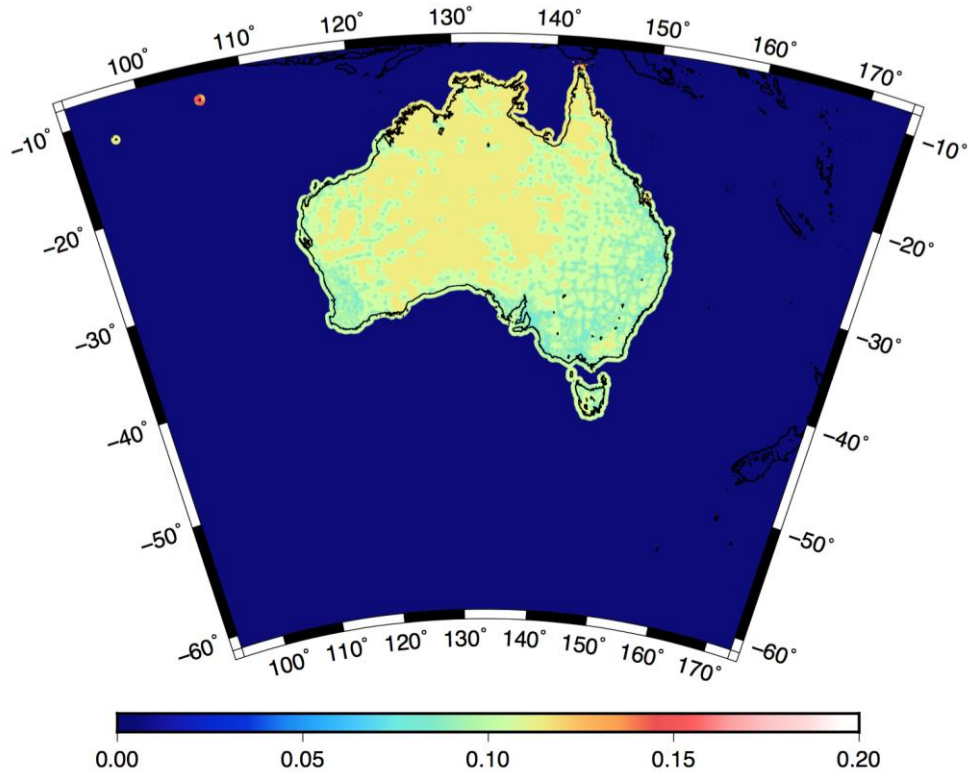


Figure 9: (a) One standard deviation of AVWS uncertainty. Units in metres. (b) One standard deviation of AUSGeoid2020 uncertainty. Units in metres from Brown et al. (2018).

The improvement in accuracy over larger distances addresses one of the biggest concerns from the users who have noticed the quality of their data (e.g. LiDAR) was starting to become more accurate than the datum (AHD) when they applied AUSGeoid. Geoscience Australia will be working with all the states and territories to continuously improve AVWS as new gravity data is included and modelling techniques are refined.

## 6.5 Computing derived AHD and AVWS heights from GNSS

AVWS heights  $H_{AVWS}^*$  can be computed by subtracting the corresponding AVWS model value from GNSS ellipsoidal height observation.

$$H_{AVWS}^* = h - \zeta_{AVWS}$$

Derived AHD heights  $H_{AHD}$  can be computed by subtracting the corresponding AUSGeoid model value from GNSS ellipsoidal height observation (Figure 10).

$$H_{AHD} = h - \zeta_{AUSGeoid}$$

NOTE: If you have GDA94 ellipsoid heights, use AUSGeoid09.

NOTE: If you have GDA2020 ellipsoidal heights, use AUSGeoid2020.

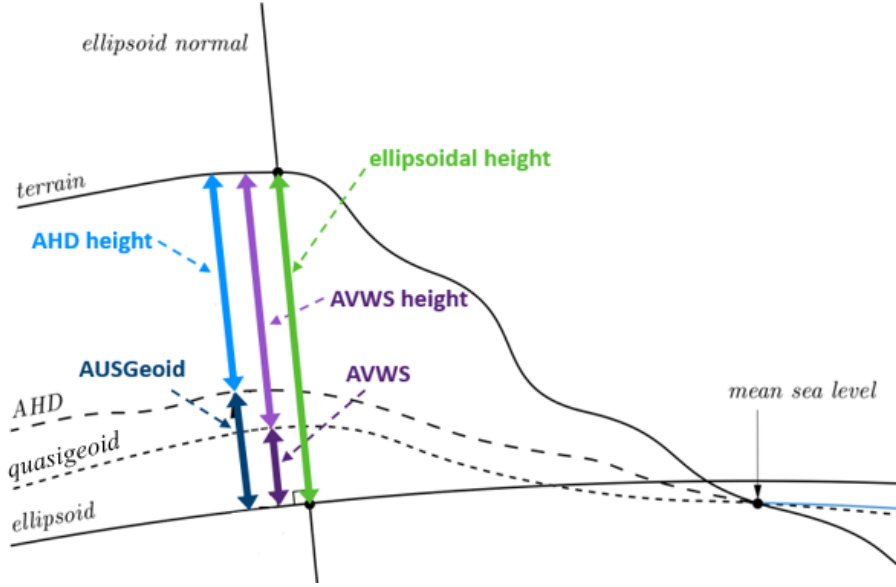


Figure 10: The AUSGeoid model (dark blue) enables users to convert ellipsoidal heights (green) to derived AHD heights (light blue). The AVWS model (dark purple) enables users to convert ellipsoidal heights (green) to AVWS heights (light purple).

## 6.6 Computing AVWS heights from levelling

To determine AVWS heights via levelling, a  $\zeta_{AVWS}$  reference point/s must first be established from GNSS height/s  $h$  and AVWS model value/s  $\zeta_{AVWS}$ .

$$H_{AVWS}^* = h - \zeta_{AVWS}$$

Heights can then be transferred via levelling. Formally, normal corrections should be applied to the relative levelling heights. The normal correction applied to levelling height differences at points A and B, is given by,

$$NC_{AB} = \sum_A^B \frac{g - \gamma_0}{\gamma_0} dn + \frac{\bar{\gamma}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{\gamma}_B - \gamma_0}{\gamma_0} H_B$$

where  $g$  are surface gravity measurements between A and B and  $\bar{\gamma}_A$  and  $\bar{\gamma}_B$  are the average normal gravity along the curved normal plumbline, between the ellipsoid and telluroid. In practice this requirements can generally be neglected at the cost of introducing a small amount of error (c.f. Filmer et al. (2010)).

For example: Suppose we have two points A at (-24.65, 153.16667) and B at (-24.6167, 115.3333) with uncorrected normal heights  $H_A = 180.8741$  and  $H_B = 181.1234$ .

The differential height of the points is  $dn = 0.2493$  m. The average gravity between the points is  $g = 9.7885607011$ . The average normal gravity of point A is  $\gamma_A = 9.7890357117$  and the average normal gravity of point B is  $\gamma_B = 9.7890125308$ . With  $\gamma_0 = 9.8061992115$  the normal gravity at 45° degrees latitude, the normal correction applied to the differential height between A and B is

$$\begin{aligned}
NC_{AB} &= \frac{g - \gamma_0}{\gamma_0} dn + \frac{\bar{\gamma}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{\gamma}_B - \gamma_0}{\gamma_0} H_B \\
&= \frac{9.7885607011 - 9.8061992115}{9.8061992115} 0.2493 \\
&\quad + \frac{9.7890357117 - 9.8061992115}{9.8061992115} \times 180.8741 \\
&\quad - \frac{9.7890357117 - 9.8061992115}{9.8061992115} \times 181.1234 = 0.0004 \text{ m}
\end{aligned}$$

## 6.7 Computing AVWS height uncertainties

Uncertainty values of heights above the AVWS,  $\sigma(H_{AVWS})$  should be modelled as the square root of the sum of Global Navigation Satellite System (GNSS) ellipsoidal height uncertainties squared,  $\sigma(h)^2$ , output from GNSS processing software and AVWS uncertainty value,  $\sigma(\zeta_{AVWS})^2$  interpolated from the AVWS uncertainty model (Featherstone et al., 2018).

$$\sigma(H_{AVWS}) = \sqrt{\sigma(h)^2 + \sigma(\zeta_{AVWS})^2} \quad (4)$$

For example:

- We have a GPS observation at [Lat: -23.6701, Long: 133.8855] with ellipsoidal height  $h = 603.244 \text{ m}$ , the standard deviation of the ellipsoidal heights after post processing is  $\sigma(h) = 0.0035 \text{ m}$ .
- The AVWS at the respective latitude and longitude is  $\zeta_{AVWS} = 15.201 \text{ m}$  and has uncertainty value  $\sigma(\zeta_{AVWS}) = 0.06$ .
- The AVWS height is then given by  $H_{AVWS} = h - \zeta_{AVWS} = 588.043 \text{ m}$
- The AVWS height uncertainty is given by  $\sigma(H_{AVWS}) = \sqrt{\sigma(h)^2 + \sigma(\zeta_{AVWS})^2} = \sqrt{0.0035^2 + 0.06^2} = \pm 0.06 \text{ m}$
- i.e. the AVWS height at our point is  $H_{AVWS} = 588.043 \pm 0.06 \text{ m}$

## 7. Access to AVWS models

The AVWS, corresponding uncertainty model, and AVWS to AHD conversion data files can be downloaded from:

AVWS quasigeoid model ( $\zeta_{AVWS}$ ):	<a href="https://s3-ap-southeast-2.amazonaws.com/geoid/AVWS/AVWS_20191107.tif">https://s3-ap-southeast-2.amazonaws.com/geoid/AVWS/AVWS_20191107.tif</a>
AVWS quasigeoid 1 sigma uncertainty model ( $\sigma(\zeta_{AVWS})$ ):	<a href="https://s3-ap-southeast-2.amazonaws.com/geoid/AVWS/AVWS_STD_20191107.tif">https://s3-ap-southeast-2.amazonaws.com/geoid/AVWS/AVWS_STD_20191107.tif</a>

To download the files, click on the link, or paste the link in an internet browser and hit Enter. The file should download automatically

Geoscience Australia is also developing an online tool to determine AVWS heights from GNSS observations (and vice versa) with  $1 \sigma$  uncertainties. This tool will be available in early 2020 and this document will be updated with a link.

## Appendix A

### A.1 The zero degree term

The Earth's gravitational potential can be represented as a spherical harmonic expansion. The zero degree term,  $\zeta_z$ , is the first spherical harmonic term. It is independent of all latitude and longitudes and can be thought of as representing constant bias in the difference between the gravity potential and the chosen reference ellipsoid. It is calculated from

$$\zeta_z = \frac{GM - GM_0}{r\gamma} - \frac{W_0 - U_0}{\gamma} \quad (A1)$$

where:

$G$  – Newton's Gravitational constant

$M$  – Mass of the Earth

$M_0$  – Mass of the ellipsoid

$\gamma$  – Normal (i.e. due to the ellipsoid) gravity at computation point

$r$  – Radius of computation point

$W_0$  – Earth potential gravity value on the geoid surface

$U_0$  – Normal (i.e. due to the ellipsoid) gravity potential on the ellipsoid

Long wavelengths (and hence the zero degree term) of AVWS come from Earth Geopotential Model 2008 (EGM2008) spherical harmonic model. The reference ellipsoid of the EGM2008 is not GRS80, but a mean-Earth ellipsoid (MEE) with estimated parameters of  $a = 6378136.58$  m and  $1/f = 298.257686$ .

#### Using EGM2008 with WGS84

The MEE is (nominally, using Eq. (A1)) ~41cm below WGS84 (Pavlis et al., 2012). Due to this offset a zero degree term (-41 cm) needs to be applied to EGM2008 quasigeoid heights to align them with WGS84.

#### Using EGM2008 with GRS80

Although, the normal gravity potential  $U_0$  value of GRS80 and WGS84 is the same, the two ellipsoids have different masses.

$$GM_{WGS84} = 3.986004418 \times 10^{14} \frac{m^2}{s^2}$$

$$GM_{GRS80} = 3.986005 \times 10^{14} \frac{m^2}{s^2}$$

Adjusting the ellipsoidal mass value in Eq. (A1) gives a  $\zeta_z$  value to align EGM2008 with GRS80,

$$\zeta_z = \frac{GM - GM_{GRS80}}{r\gamma} - \frac{W_0 - U_0}{\gamma} = -0.41 \text{ m} + \frac{GM_{WGS84} - GM_{GRS80}}{r\gamma} = -0.41 \text{ m} +$$
$$-0.93 \text{ m} = -1.34 \text{ m}$$

The need to go from EGM2008 to WGS84 and then from WGS84 to GRS80 rather than from EGM2008 to GRS80 directly is because there is not information published about the EGM2008 parameters shown in A1.

To ensure AVWS can operate in harmony with GDA2020 and ATRF the -1.34 m bias has been applied to the AVWS model, so that it is aligned with GRS80 to be directly compatible with GNSS observations.

*NOTE: The Australian Gravimetric Quasigeoid 2017 (AGQG2017) model is EGM2008 enhanced at high spatial frequencies, using gravity data which are available across the Australian continent. Since at long wavelengths AGQG2017 and EGM2008 are the same, the reference ellipsoid of AGQG2017 the same mean-Earth ellipsoid used for EGM2008. This means there is a 41 cm difference between AGQG2017 and AVWS.*

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