Tidal Planes and MSL Errors

Nicholas Dando and Bill Mitchell, Australia

Key words: Hydrography, tidal error model

SUMMARY

The Australian tidal error model is the first attempt to define uncertainties in Mean Sea Level (MSL) around the Australian coastline. Tide gauge observations for the Australian coast span from less than 1 month to greater than 100 years. The high quality, decade and longer observation tide gauges are used in the production of the frequency dependant error surface. The observed hourly data are analysed using the National Tidal Centre TANS analysis package, to estimate harmonic constituents (used for prediction), a MSL determination, a fitted linear trend and a residual. The power spectrum of the residual is then separated into a predefined set of frequency bins, representing the noise levels of the sea surface proportional to frequency. The longer span observations fill more of this predefined spectrum, specifically the lower frequency errors, which contribute a significant proportion to the error. Spatial interpolation around Australia is performed individually for each frequency bin. This method allows regions with shorter spans of observations to have lower frequency error added, creating a synthetic spectrum at the interpolation point. These synthetic spectrums are then used to determine confidence intervals of MSL around the coastline of Australia. The understanding of these errors is an important step for combining bathymetry and topography datasets, ultimately creating a seamless national digital elevation model.

CONTACT

Nicholas Dando
National Geodetic Reference Systems Project
Geospatial & Earth Monitoring Div
Geoscience Australia
GPO Box 378
Canberra
2601
ACT
Australia
+ 61 2 6249 9552
Nicholas.Dando@ga.gov.au

Mean Sea Level tidal plane errors

CRC-SI Project 1.14:
"Reconciling Australia Height datum's: the Vertical component"

Nicholas Dando

Geoscience Australia
National Geospatial Reference Systems Project
Email: Nicholas.Dando@ga.gov.au
PO Box 378, Canberra,
ACT 2601, Australia

Bill Mitchell

National Tidal Centre (NTC)
Bureau of Meteorology (BOM)
Email: B.Mitchell@bom.gov.au
South Australian Office
25 College Rd. Kent town
PO Box 421. SA 5071

Co-operative Research Centre for Spatial Information

Australian Government – Geoscience Australia

Australian Government – Bureau of Meteorology

XXIV FIG International Congress 2010, Sydney

The problem:

Bathymetry and Topography datums don't match:

- Bathymetry datum is CD/LAT
  - Bathymetry measured from MSL observations & adjusted down to LAT/CD
  - Varying length MSL obs. from days to decades in length
  - Varying length MSL obs. give inconsistent datums between bathymetry datasets
- Topography datum is AHD
  - AHD referenced to 1966 - 1968 MSL

Solution:

- Define a common datum with uncertainties for varying span obs.

Overview:

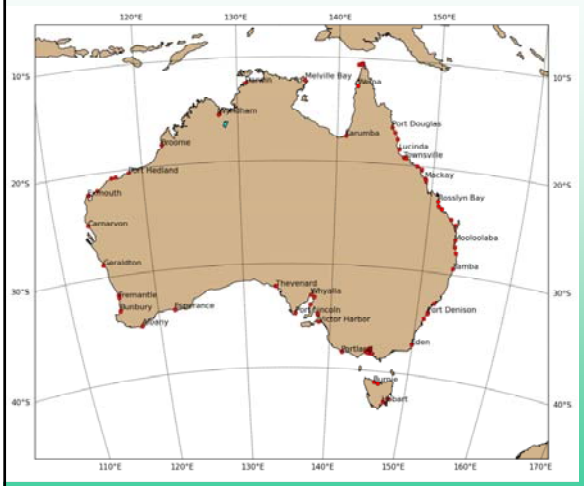
Method for comparing various obs. spans of MSL:

1. Select long term tide gauges for residual baseline (defining MSL datum).
   - At these gauges uncertainty = 0.000m
2. Get periodogram amplitude estimates (spectrum) of residual via Fourier Transform
3. Interpolate spectrum components to get synthetic spectrum at short span MSL obs. point.
4. Synthetic spectrum modified for short obs. span uncertainty, relative to long term baseline.

Later project/further work:

- Perform method on all varying span MSL obs. with ellipsoidal heights, collate for MSL uncertainty surface.

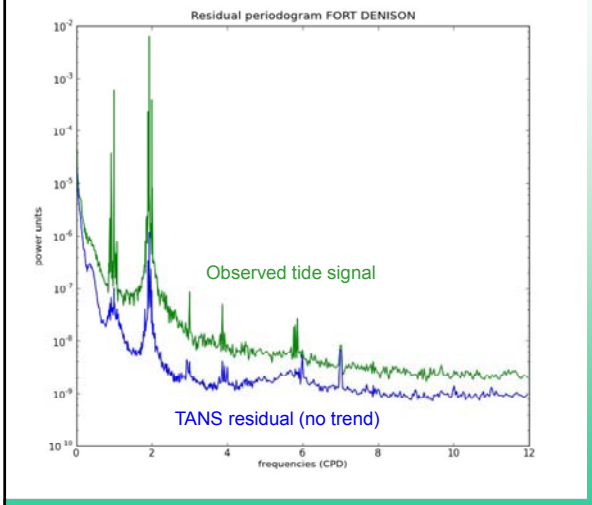
Data



- Modern 19 yr (1989-2007) data epoch
- Tides removed using TANS[1] analysis
  - Astronomical energy removed
- Spectrum of residuals used for uncertainties

1. TANS: National Tidal Centre's in house tidal analysis package

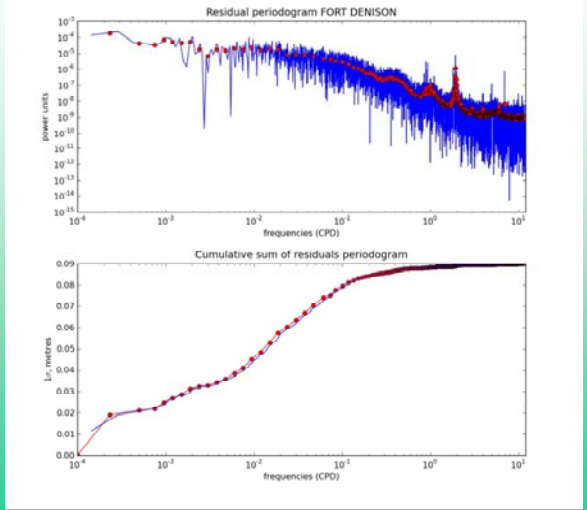
Data



- Modern 19 yr (1989-2007) data epoch
- Tides removed using TANS[1] analysis
  - Astronomical energy removed
- Spectrum of residuals used for uncertainties

1. TANS: National Tidal Centre's in house tidal analysis package

Tidal residuals



- Tide gauges have unique meteorological noise
- Parseval's theorem > error from spectrum.

  Spectrum sum = variance of signal

  $x[n]$ tidal residual

  $X[f]$ residual spectrum

$$\sum_{n=0}^{N-1}|x[n]|^2 = \frac{1}{N}\sum_{n=0}^{N-1}|X[f]|^2$$

- Spectrums redistributed (& smoothed) into 500+ bins $R[f]$ for comparison and interpolation.

Red dots: 500+ redistributed spectrum bins $R[f]$

Interpolating spectrum

- Interpolated baseline spectrum components to create synthetic spectrum at interpolation point.
- Assumption: Low freq. spectrum has high spatial correlation. (wide bands not individual frequencies)
- Interpolation performed for each redistributed spectrum bin *R[f]*
- Inverse distance weighting used (Shepard, 1968)
  - Fits data points *R[f]*, only 66 points
  - Copes with irregular data points.
  - Very simple, smoother than linear interpolation

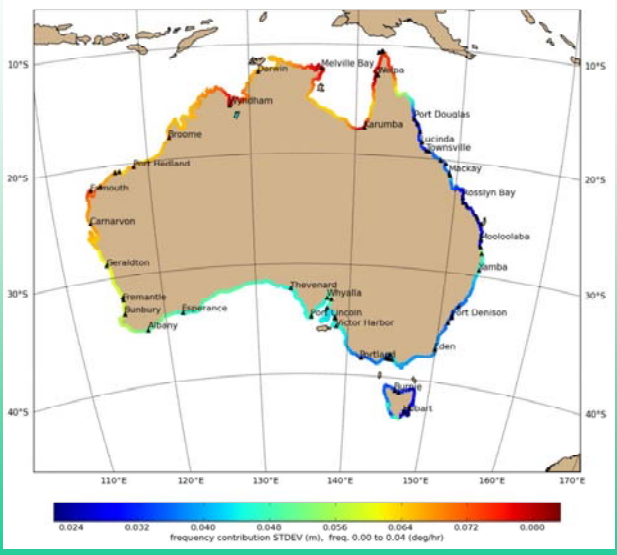
Shepard's inverse distance weighting:

$$S[x] = \sum \frac{w_k(x)}{\sum w_k(x)} \cdot R[f]$$

$$w_k(x) = \frac{1}{d(x, x_k)^p}$$

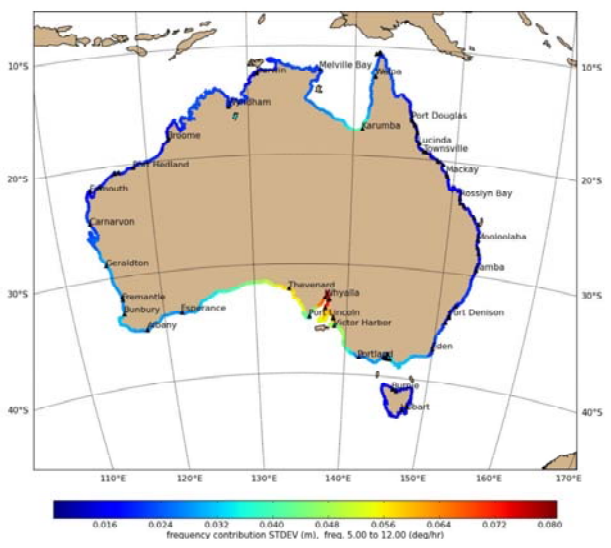
x	interpolation point
$w_k(x)$	weighting function
$S[x]$	synthetic spectrum value
$R[f]$	spectrum component
$d(x, x_k)$	distance
p	proponent = 2

Frequency based error



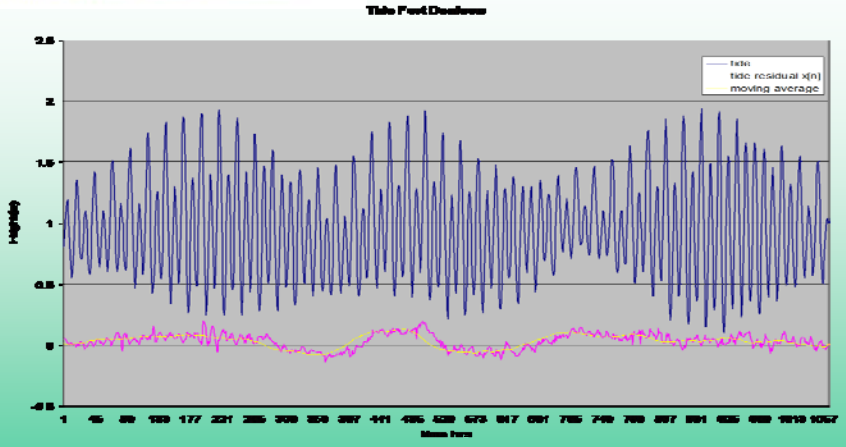
- Frequency contribution (1σ) in periods of 1 – 19yrs
- High error in northern regions > 0.06m
- Low error ~0.02m behind great barrier reef.

Frequency based error



- Frequency contribution (1σ) of 3 days – 19.2hrs frequencies
- High error >0.06m in upper reaches of Spencer Gulf
- Low error < 0.024m most of coastline

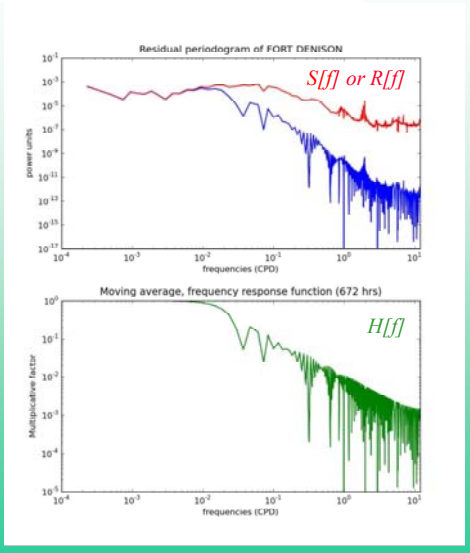
Moving average filter



Time domain: A moving average (MA) filter applied to tidal residual $x[n]$ is a 'signal' of short span MSL's obs. RMS (standard deviation) of this signal is uncertainty of short span MSL obs. to long term average.

RMS (standard deviation) can also be calculated from spectrum by applying filter in frequency domain to calculate uncertainty of 'synthetic signal' by modifying synthetic spectrum.

Modified spectrum



- Modify synthetic spectrum with MA filter frequency response function

MA frequency response function:

$$H[f] = \frac{\sin(\pi f M)}{M \sin(\pi f)}$$

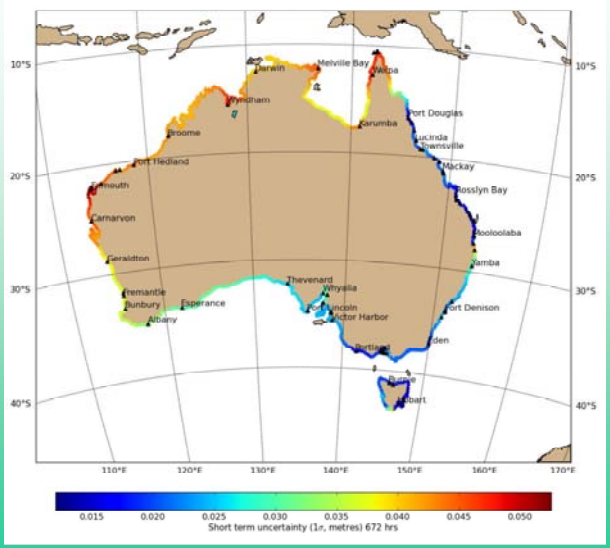
$H[f]$	Amp. factor
M	hrs obs.
f	freq (cycles/sample)

Uncertainty (1σ) of short span MSL:

$$\sigma = \sqrt{\frac{1}{N}\sum_{f=0}^{N-1} |S[f] \times H[f]|^2}$$

$S[f]$	Synthetic spectrum frequency estimate

Short span obs. uncert.



Theoretical uncertainty of short span 28 day MSL obs. around coastline

- Modified spectrum is unique to each short span MSL obs.
- Repeated for all short span obs.
- 28 day moving average filter (672 hrs)
- Broadly similar to low freq. error
- Long period tides corrected or added back.

Summary

- Baseline long term tide gauges used as MSL datum
- Synthetic spectrums are calculated around the coast.
- Synthetic spectrums are modified to produce uncertainty at short span MSL obs. points
- With ellipsoidal heights MSL observations around coastline can be greatly expanded.
- Method adaptable for AHD and older sea levels comparison.

Thank you

Australian Government
Geoscience Australia

Nicholas Dando
National Geospatial Reference Systems Project
Ph: +61 2 6249 9552
Fax: +61 2 6249 9929
Email: Nicholas.Dando@ga.gov.au

Geoscience Australia
PO Box 378, Canberra, ACT 2601, Australia