ANZLIC COMMITTEE ON SURVEYING & MAPPING

Geocentric Datum of Australia

Technical Manual

Version 2.4

ISBN 0-9579951-0-5

Intergovernmental Committee on Surveying and Mapping (ICSM)

Permanent Committee on Geodesy (PCG)

2 December 2014

Document History

DATE	VERSION	AMENDMENTS
Dec 2014	2.4	Update to foreword Copyright statement updated Typographical changes Chapter 1 – gazettal values updated and text refreshed for (non) equivalency of GDA94 and WGS84/ITRF Chapter 3 – Figures removed Chapter 7 – Similarity Transformation matrix corrected Chapter 9 – AUSGeoid09 update, example data updated, equation transcription errors corrected
Jul 2009	2.3(1)	Typographical changes, layout and hyperlink amendments
Feb 2006	2.3	Chapter 1 – Modified statement re GDA and ITRF Chapter 7 – Modified redirection to ITRF and GDA information Chapter 9 – AUSGeoid to AHD explained in greater detail
Feb 2003	2.3	Chapter 7 – Corrected Error from October 2001 rewrite, Page 7-32 Combined Rotation Matrix
Feb 2002	2.2	Converted to PDF format
Oct 2001	2.2	Chapter 7 – Comparison of transformation methods – updated to use national NTv2 grids and national AGD66 similarity parameters; Similarity transformation – AGD66 national parameters added; amendments made to include the National Transformation Grids and test data for them. Chapter 9 – More detail added on the relationship between AHD and the geoid Bibliography – added paper by Collier and Steed on transformation grids Chapter 1 – amended to include references to ITRF/GDA94 transformation

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Terms and Definitions

Item	Symbol	Explanation
Semi-major axis	a	Ellipsoid semi-major axis.
Semi-minor axis	b	Ellipsoid semi-minor axis: $b = a(1-f)$.
Flattening	f	The relationship between the semi-major and semi-minor axes of the ellipsoid: $(a - b)/a$.
Inverse flattening	$1/f$	The reciprocal of the ellipsoid flattening. This is the value commonly used when specifying an ellipsoid (e.g. 1/f = 298.257).
Eccentricity squared	e^2	$(a^2 - b^2)/a^2$
Second eccentricity squared	e'^2	$(a^2 - b^2)/b^2$
Radius of curvature	ρ	Radius of curvature of the ellipsoid in the plane of the meridian.
	ν	Radius of curvature of the ellipsoid in the prime vertical.
	R	Geometric mean radius of curvature: $(\rho\nu)^{1/2}$.
	R_α	Radius of curvature at a point, in a given azimuth. It may vary by thousands of metres, depending on the azimuth.
	Ψ	Ratio of the ellipsoidal radii of curvature: ν/ρ.
	r^2	$R^2\, k_0{}^2 = \rho\nu k_0{}^2$
	$r_m{}^2$	$\rho\nu k_0{}^2$ at Φ_m
Latitude	Φ	Geodetic latitude, negative south of the equator.
	Φ_1, Φ_2	Geodetic latitude at points 1 and 2 respectively.
	Φ_m	Mean latitude: $(\Phi_1 + \Phi_2)/2$.
	$\Delta\Phi$	Latitude difference: $\Phi_2 - \Phi_1$.
Foot point latitude	Φ'	Latitude for which the meridian distance $(m) = N'/k_0$. t', Ψ', ρ', ν' are functions of the latitude Φ'.
Longitude	λ	Geodetic longitude measured from Greenwich, positive eastwards.
	Λ_1, λ_2	Geodetic longitude at points 1 and 2 respectively.
	$\Delta\lambda$	Longitude difference: $\lambda_2 - \Lambda_1$.
	λ_0	Geodetic longitude of the central meridian.
	ω	Geodetic longitude difference measured from the central meridian, positive eastwards: $\lambda - \lambda_0$.
Azimuth	α	Horizontal angle measured from the ellipsoidal meridian, clockwise from north through 360°.

Ellipsoidal distance	s	Distance on the ellipsoid along either a normal section or a geodesic. The difference between the two is usually negligible, amounting to less than 20 millimetres in 3,000 kilometres. A line on the ellipsoid is projected on the grid as an arc.
Sea level or geoidal distance	s'	Distance reduced using heights above sea level or the geoid, which are often referred to as orthometric heights. Ellipsoidal distances should be used for GDA computations.
Easting	E'	Measured from a Central Meridian, positive eastwards.
	E	Measured from the false origin (E' + 500,000 metres for MGA94).
Northing	N'	Measured from the equator, negative southwards.
	N	Measured from the false origin (N' in the northern hemisphere; N' + 10,000,0000 metres in the southern hemisphere for MGA94).
Grid convergence	γ	Angular quantity to be added algebraically to an azimuth to obtain a grid bearing: Grid Bearing = Azimuth + Grid Convergence. In the southern hemisphere, grid convergence is positive for points east of the central meridian (grid north is west of true north) and negative for points west of the central meridian (grid north is east of true north).
Grid Bearing	β	Angle between grid north and the tangent to the arc at the point. It is measured from grid north clockwise through 360°.
Arc-to-chord Correction	δ	Angular quantity to be added algebraically to a grid bearing to obtain a plane bearing: $\theta = \beta + \delta = \alpha + \gamma + \delta$. The arc-to-chord corrections differ in amount and sign at either end of a line. Lines that do not cross the central meridian always bow away from the central meridian. In the rare case of a line that crosses the central meridian less than one-third of its length from one end, the bow is determined by the longer part. Note that $\Delta\beta = \delta_1 - \delta_2$ and the sign is defined by the equations: $\theta = \beta + \delta = \alpha + \gamma + \delta$. The arc-to-chord correction is sometimes called the 't-T' correction.
Meridian convergence	$\Delta\alpha$	The change in the azimuth of a geodesic between two points on the spheroid: Reverse Azimuth = Forward Azimuth + Meridian Convergence $\pm$ 180°: $\alpha_{21} = \alpha_{12} + \Delta\alpha \pm 180°$.
Line curvature	$\Delta\beta$	The change in grid bearing between two points on the arc. Reverse grid bearing = Forward grid bearing + Line curvature $\pm$ 180°: $\beta_2 = \beta_1 + \Delta\beta \pm 180°$.

Plane bearing	θ	The angle between grid north and the straight line on the grid between the ends of the arc formed by the projection of the ellipsoidal distance; measured clockwise through 360°.
Grid distance	S	The length measured on the grid, along the arc of the projected ellipsoid distance.
Plane distance	L	The length of the straight line on the grid between the ends of the arc of the projected ellipsoidal distance. The difference in length between the plane distance (L) and the grid distance (S) is nearly always negligible. Using plane bearings and plane distances, the formulae of plane trigonometry hold rigorously: $\tan\theta = \Delta E/\Delta N$, $\Delta E = L\sin\theta$, $\Delta N = L\cos\theta$.
Meridian distance	m	True distance from the equator, along the meridian, negative southwards.
	G	Mean length of an arc of one degree of the meridian.
	σ	Meridian distance expressed as units G: $\sigma = m/G$.
Central scale factor	k_0	Scale factor on the central meridian (0.9996 for MGA94).
Point scale factor	k	Ratio of an infinitesimal distance at a point on the grid to the corresponding distance on the spheroid: $k = dL/ds = dS/ds$ It is the distinguishing feature of conformal projections, such as the Universal Transverse Mercator used for MGA94, that this ratio is independent of the azimuth of the infinitesimal distance.
Line scale factor	K	Ratio of a plane distance (L) to the corresponding ellipsoidal distance (s): $K = L/s \approx S/s$. The point scale factor will in general vary from point to point along a line on the grid.
Ellipsoidal height	h	Ellipsoidal Height (h) is the distance of a point above the ellipsoid, measured along the normal from that point to the surface of the ellipsoid used.
	Δh	Change in ellipsoidal height (m).
Height above the geoid	H	Height of a point above the geoid measured along the normal from that point to the surface of the geoid. It is also referred to as the orthometric height.
	H_{AHD}	The derived difference in height, from AUSGeoid09, between AHD and the surface.
Geoid-ellipsoid separation	N	Distance from the surface of the ellipsoid used, to the surface of the geoid measured along the normal to this ellipsoid. This separation is positive if the geoid is above the ellipsoid and negative if the geoid is below the ellipsoid: $h - H =$ geoid ellipsoid separation.

	N_{AHD}	The distance between the ellipsoid and AUSGeoid09 measured along the normal to this ellipsoid.
Earth-centred Cartesian coordinates.	X, Y, Z	A three dimensional coordinate system which has its origin at (or near) the centre of the earth. These coordinates are commonly used for satellite derived positions (e.g. GNSS) and although they relate to a specific reference system they are independent of any ellipsoid. The positive Z axis coincides with (or is parallel to) the earth's mean axis of rotation and the X and Y axes are chosen to obtain a right-handed coordinate system; for convenience it can be assumed that the positive arm of the X axis passes through the Greenwich meridian.
Transformation parameters	Δa	Change in ellipsoid semi-major axis (e.g. from ANS to GRS80) (m).
	Δf	change in ellipsoid flattening (e.g. from ANS to GRS80).
	ΔX	origin shift along the X axis (m).
	ΔY	origin shift along the Y axis (m).
	ΔZ	origin shift along the Z axis (m).
	R_x	Rotation of the X axis (radians); positive when anti-clockwise as viewed from the positive end of the axis looking towards the origin.
	R_y	Rotation of the Y axis (radians); positive when anti-clockwise as viewed from the positive end of the axis looking towards the origin.
	R_z	Rotation of the Z axis (radians); positive when anti-clockwise as viewed from the positive end of the axis looking towards the origin.
	Sc	Change in scale (parts per million – ppm).

Foreword

The Geocentric Datum of Australia Technical Manual is principally designed to explain all facets of the Geocentric Datum of Australia, and continues the tradition of providing complete formulae and worked examples.

To cater for the enormous changes that have taken place since the Australian Geodetic Datum Technical Manual was originally published, the chapters on the geoid and coordinate transformation have been expanded. A brief history of Australian coordinates has also been included.

The coordinates of the Australian Fiducial Network (AFN) geodetic stations have been updated to reflect the 2012 gazettal of 21 geodetic stations, strengthening and densifying Australia's recognized-value standard for position.

Chapter 1 Background and Explanation

The Geocentric Datum of Australia (GDA) is the Australian coordinate system, replacing the Australian Geodetic Datum (AGD). GDA is part of a Global Geodetic Reference Frame (GGRF) and is directly compatible with Global Navigation Satellite Systems (GNSS) such as the Global Positioning System (GPS), GLObal NAvigation Satellite System (GLONASS), BeiDou, and Galileo, as well as Regional Navigation Satellite Systems (RNSS), such as the Indian Regional Navigational Satellite System (IRNSS) and Quasi-Zenith Satellite System (QZSS). It is the responsibility of the Intergovernmental Committee on Surveying and Mapping (ICSM) to maintain GDA.

Background to GDA

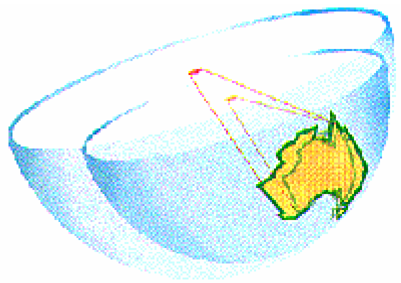


Figure 1-1: Changing datums

In 1992, as part of the world-wide International GNSS Service (IGS) campaign, previously known as the International GPS Service, continuous GPS observations were undertaken on eight geologically stable marks at sites across Australia, which formed the Australian Fiducial Network (AFN) – the Recognized-value standard of measurement of position. During this campaign, GPS observations were also carried out at a number of existing geodetic survey stations across Australia. These were supplemented by further observations in 1993 and 1994, producing a network of about 70 well determined GNSS sites, with nominal 500 km spacing across Australia. These sites are collectively known as the Australian National Network (ANN).

The GPS observations at both the AFN and ANN sites were combined in a single regional GPS solution in terms of the International Terrestrial Reference Frame 1992 (ITRF92) and the resulting coordinates were mapped to a common epoch of 1994.0. The positions for the AFN sites were estimated to have an absolute accuracy of about 2 cm at 95% confidence (Morgan, Bock et al. 1996), while the ANN positions are estimated to have an absolute accuracy of about 5 cm. These positions of the AFN sites were used to define the Geocentric Datum of Australia (GDA) and were published in the Commonwealth of Australia Government Gazette on 6 September 1995.

In 2012, the AFN was updated with new coordinates and to include 21 sites. The purpose of the update was to improve its consistency with the most recent realisation of the International Terrestrial Reference Frame. The updated AFN coordinates have been adopted from ITRF2008 and subsequently transformed to GDA94 (i.e. ITRF1992 at epoch 1994.0) using the Dawson-Woods transformation parameters (2010). For those stations with multiple coordinate estimates in ITRF 2008 the most recent coordinate estimate has been adopted.

The new Gazettal values are in shown in the Commonwealth of Australia Gazette extract below.

Commonwealth of Australia Gazette

No. GN 1 4 April 2012 Government Departments

GEOCENTRIC DATUM FOR AUSTRALIA

The meeting of the Intergovernmental Committee on Surveying and Mapping held in Canberra on 28-29 November 1994 adopted the following geodetic datum for Australia and recommended its progressive implementation Australia-wide by 1 January 2000:

Designation – The Geocentric Datum of Australia (GDA)

Reference Ellipsoid – Geodetic Reference System 1980 (GRS80) ellipsoid with a semi-major axis (a) of 6 378 137 metres exactly and an inverse flattening (1/f) of 298.257 222 101

Reference Frame – The GDA is realised by the co-ordinates of the following Australian Fiducial Network (AFN) geodetic stations referred to the GRS80 ellipsoid determined within the International Earth Rotation Service Terrestrial Reference Frame 1992 (ITRF92) at the epoch of 1994.0:

Peter Fisk

Chief Metrologist, National Measurement Institute

Location (Identifier)	East Longitude			South Latitude			Elevation (from ellipsoid)
Adelaide (ADE1)	138°	38′	50.43453″	34°	43′	44.41404″	38.1293 m
Alice Springs (ALIC)	133°	53′	7.84785″	23°	40′	12.44602″	603.3466 m
Burnie (BUR1)	145°	54′	53.4412″	41°	3′	0.24611″	3.1531 m
Ceduna (CEDU)	133°	48′	35.37558″	31°	52′	0.01657″	144.8196 m
Cocos Island (COCO)	96°	50′	2.27360″	12°	11′	18.06739″	-35.2309 m
Darwin (DARW)	131°	7′	57.8479″	12°	50′	37.35864″	125.2124 m
Hobart (HOB2)	147°	26′	19.43574″	42°	48′	16.98538″	41.1389 m
Karratha (KARR)	117°	5′	49.87277″	20°	58′	53.16983″	109.2269 m
Macquarie Island (MAC1)	158°	56′	8.99643″	54°	29′	58.32845″	-6.7231 m
Melbourne (MOBS)	144°	58′	31.20646″	37°	49′	45.89904″	40.6710 m
New Norcia (NNOR)	116°	11′	33.76803″	31°	2′	55.46714″	234.8939 m
Parkes (PARK)	148°	15′	52.58905″	32°	59′	55.58175″	397.4436 m
Perth (PERT)	115°	53′	6.88639″	31°	48′	7.09686″	12.7697 m
Mount Stromlo (STR1)	149°	0′	36.17971″	35°	18′	55.93958″	800.0226 m
Mount Stromlo (STR2)	149°	0′	36.54756″	35°	18′	58.1991″	802.5632 m
Sydney (SYDN)	151°	9′	1.35691″	33°	46′	51.18426″	85.6781 m
Tidbinbilla (TIDB)	148°	58′	47.98444″	35°	23′	57.15619″	665.4232 m
Townsville (TOW2)	147°	3′	20.46531″	19°	16′	9.42824″	88.2292 m
Christmas Island (XMIS)	105°	41′	18.58058″	10°	26′	59.89868″	261.5892 m
Yarragadee (YAR1)	115°	20′	49.10002″	29°	2′	47.61682″	241.3533 m
Yarragadee (YARR)	115°	20′	49.08938″	29°	2′	47.74264″	241.4231 m

Figure 1-2: Extract from Commonwealth of Australia Gazette, available:
http://www.comlaw.gov.au/Details/F2012L00800

GDA Specifications

Terminology

Datum: Geocentric Datum of Australia (GDA)

Geographical coordinate set (latitude and longitude): Geocentric Datum of Australia 1994 (GDA94)

Grid coordinates (Universal Transverse Mercator, using the GRS80 ellipsoid): Map Grid of Australia 1994 (MGA94)

Definition

Reference Frame	ITRF92 (International Terrestrial Reference Frame 1992)
Epoch	1994.0
Ellipsoid	GRS80
Semi-major axis (a)	6,378,137.0 m
Inverse flattening (1/f)	298.257222101

GDA Extent

Includes all the areas contained within Australia's marine jurisdiction within 200 nautical miles of Australia and its external territories, and the areas of Australia's continental shelf beyond 200 nautical miles as confirmed by the United Nations Commission on the Limits of the Continental Shelf. The areas include Cocos (Keeling) Island, Christmas Island, Norfolk Island and Macquarie Island but excludes Heard-McDonald Island and the Australian Antarctic Territory (AAT) as shown in Figure 1-3.

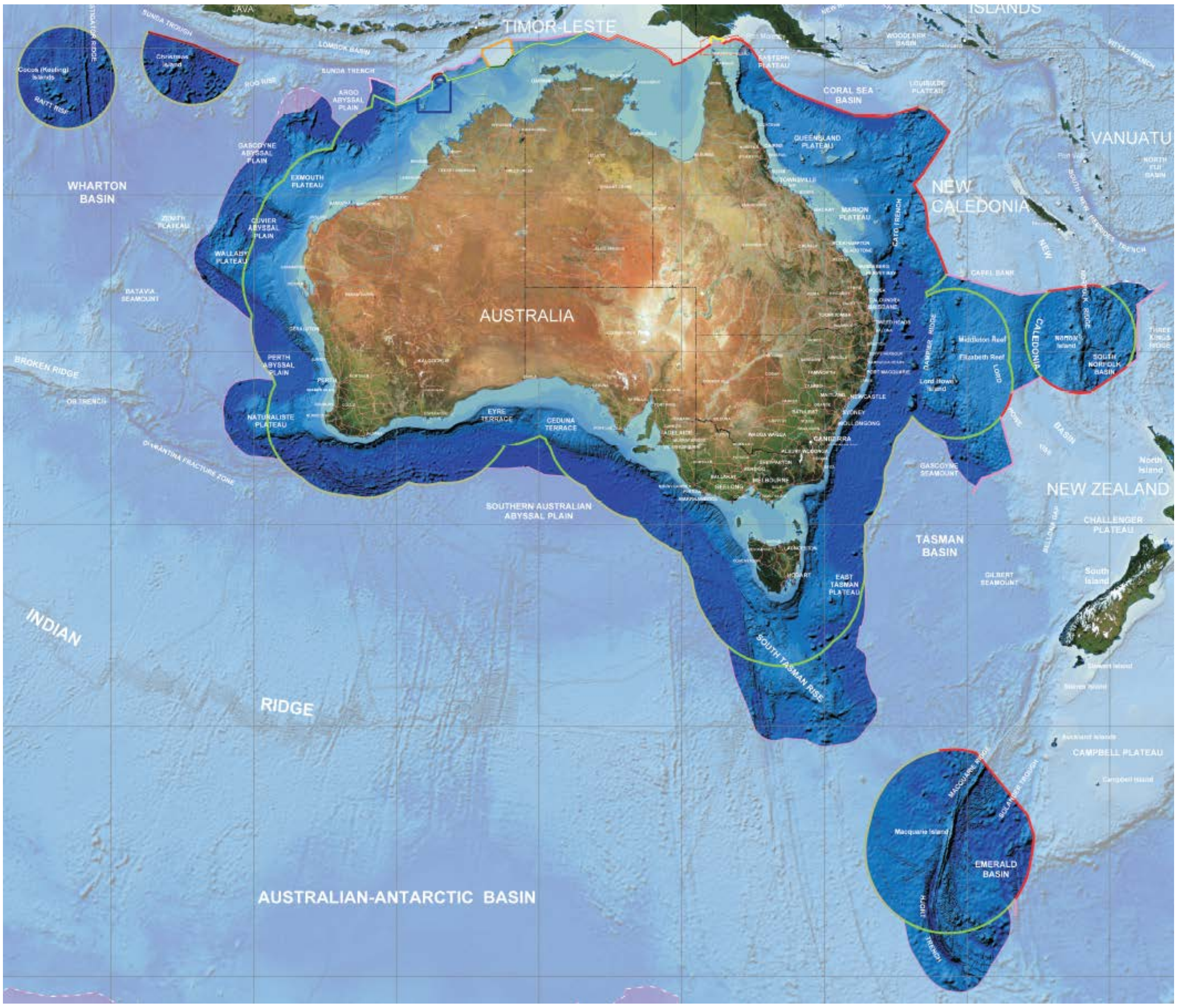


Figure 1-3: The area shown in dark blue is the GDA94 extent. The colours of the lines represent different types of jurisdictional boundaries or proposed jurisdictional boundaries. For more information on the type of boundary, please refer to http://www.ga.gov.au/metadata-gateway/metadata/record/gcat_70362.

GDA and AGD

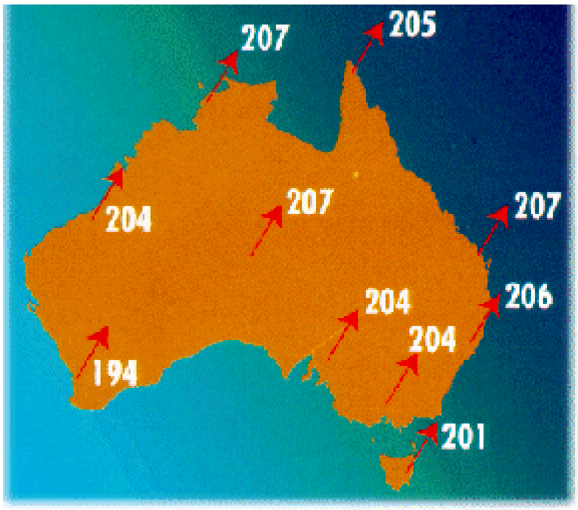


Figure 1-4: Difference between AGD and GDA94 coordinates

ITRF92, on which GDA is based, was realised using Very Long Baseline Interferometry (VLBI), GPS and Satellite Laser Ranging (SLR) observations at 287 globally distributed stations (Boucher, Altamimi et al. 1993). However, the coordinates for Johnston, the origin station for AGD, were based on a selection of 275 astro-geodetic stations distributed over most of Australia (Bomford 1967).

The adoption of this origin and the best fitting local ellipsoid, the Australian National Spheroid (ANS), meant that the centre of the ANS did not coincide with the centre of mass of the earth, but lay about 200 metres from it. Hence, the GDA94 coordinates of a point appear to be about 200 metres north east of the AGD coordinates of the same point.

The precise size and orientation of the difference will vary from place to place. More detailed information, including methods of transformation, is available in Chapter 7.

GDA, ITRF and WGS84

The Geocentric Datum of Australia is a realisation of the International Terrestrial Reference Frame 1992 (ITRF92) at epoch 1994.0. ITRF is a global network of accurate coordinates (and their velocities) maintained by the International Earth Rotation Service (IERS) and derived from geodetic observations (VLBI, SLR, GPS and DORIS [Doppler Orbitography and Radio positioning Integrated by Satellite]) (Seeber 1993).

The World Geodetic System, of which the latest revision is WGS84, is the datum used by the GPS operated by the U.S. Department of Defense. The datum is defined and maintained by the United States National Geospatial-Intelligence Agency (NGA). WGS84 has been revised several times since its conception and is at present aligned at the centimetre level to the ITRF (NGA, 2014), which generally ensures scientific integrity and compatibility with international standards and conventions. The WGS84 coordinates of tracking stations used to compute the GPS broadcast orbit are adjusted annually for plate tectonic motion to an epoch at the half year mark, e.g. WGS84 as used in the GPS broadcast orbit during calendar year 2014 is ITRF2008@2014.5. Consequently, differences between the ITRF and WGS84 are negligible for most users.

In January 1994 GDA94 and ITRF were coincident, but as the Australian tectonic plate is moving at about 7 cm per year in a north easterly direction there is an increasing difference in positions in terms of the two systems. This will amount to about 1.8 m difference between the two systems by 2020. For applications that require uncertainty better than 5 m, GDA94 and WGS84/ITRF cannot be considered as equivalent and users should apply Dawson and Woods (2010) methodology to transform coordinates between GDA94 and WGS84/ITRF.

The ellipsoid recommended by the International Association of Geodesy (IAG) and used with the GDA, is the Geodetic Reference System 1980 ellipsoid. The parameters of the WGS84 ellipsoid "... *are identical to those for the GRS80 ellipsoid with one minor exception. The coefficient form used for the second degree zonal is that of the WGS84 Earth Gravitational Model rather than the notation J2 used with GRS80*." (DMA 1987). Consequently, "*the GRS80 and WGS84 ellipsoids have a very small difference in the inverse flattening, but this difference is insignificant for most practical applications*".

Ellipsoid	GRS80	WGS84
Semi major axis (a)	6,378,137.0	6,378,137.0
Inverse flattening (1/f)	298.257222101	298.257223563

Grid Coordinates

Geodetic coordinates (latitude and longitude) are represented on a map or chart, by mathematically "projecting" them onto a surface, which can be laid flat.

The Transverse Mercator system projects geodetic coordinates onto a concentric cylinder which is

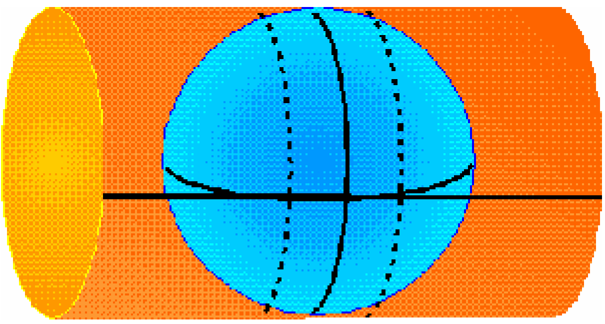


Figure 1-5: Transverse Mercator projection

tangent to the equator and makes contact along one meridian.

To minimise distortion, the earth is "rotated" within the cylinder, to bring a different meridian into contact with the cylinder, for different areas. This results in north-south bands known as zones. The true origin for each zone is the intersection of the equator and the contacting meridian (the central meridian), but a false origin is often used to avoid negative coordinates. In 1947, the US Army adopted uniform scale factor, false origins and zone size and numbering for the TM projection and these have since been generally accepted as the Universal Transverse Mercator Projection (UTM) (Snyder 1984). This projection was used with the Australian National Spheroid and AGD66 and AGD84 latitudes and longitudes to produce the Australian Map Grid 1966 and Australian Map Grid 1984 coordinates (AMG66 and AMG84). It is also used with the GRS80 ellipsoid and GDA94 latitudes and longitudes to produce Map Grid of Australia 1994 coordinates (MGA94).

Redfearn's formulae (Chapter 5) are used to convert between UTM and geodetic coordinates.

Table 1-1: UTM Parameters

Longitude of initial central meridian (Zone one)	177 degrees west longitude
Zone width	6 degrees
Central scale factor	0.9996
False easting	500,000 m
False northing (in the southern hemisphere)	10,000,000 m

Other Coordinates used in Australia

With the introduction of AGD in 1966, AGD66 coordinates were widely adopted but were later replaced in several States by the improved AGD84 coordinates. However there were also a number of early global coordinate systems, which were used mainly with satellite navigation systems (Steed 1990).

Australian Geodetic Datum (AGD)

The Australian Geodetic Datum was the first proclaimed in the Australian Commonwealth Gazette of 6 October 1966. This proclamation included the parameters of the adopted ellipsoid, known as the Australian National Spheroid (ANS), and the position of the origin point – Johnston Geodetic Station.

The coordinates (latitude & longitude) produced by the 1966 national adjustment in terms of the AGD are known as AGD66 and the equivalent UTM grid coordinates are known as AMG66.

In 1982 a new national adjustment, referred to as the Geodetic Model of Australia 1982 (GMA82), was performed using all data previously included in the 1966 adjustment as well as more recent observations. This new adjustment used the same gazetted AGD values as the AGD66 adjustment, but used improved software and included a geoid model. The coordinates resulting from this adjustment were accepted by the National Mapping Council in 1984 and are known as Australian Geodetic Datum 1984 (AGD84) coordinates. The equivalent UTM grid coordinates are known as AMG84.

Table 1-2: ANS Ellipsoid Parameters

Semi major axis (a)	6,378,160 metres
Inverse Flattening (1/f)	298.25

World Geodetic System 1972 (WGS72)

WGS72 was the third approximately geocentric reference frame developed by the United States Defense Mapping Agency (DMA) to support its activities (previous versions were WGS60 and WGS66). It was superseded by WGS84, but until 27 January 1987, was used with the GPS system and prior to 27 January 1989 it was used for the Transit Doppler navigation system broadcast ephemeris. In the Australian region, WGS72 coordinates differ from WGS84 and GDA94 coordinates by about 15 metres.

Table 1-3: WGS72 Ellipsoid Parameters

Semi major axis (a)	6,378,135 metres
Inverse Flattening (1/f)	298.26

NSWC-9Z2

This system, which was effectively the same as its predecessor NWL9D, was an approximately geocentric system used for the Transit Doppler navigation system "precise" ephemerides.

Table 1-4: NSWC-9Z2 Ellipsoid Parameters

Semi major axis (a)	6,378,145 metres
Inverse Flattening (1/f)	298.25

"Clarke" Coordinates

In Australia prior to 1966, some twenty different datums, using four different ellipsoids were used. The most widely used was the Clarke's 1858 ellipsoid:

Table 1-5: Clarke 1858 Ellipsoid Parameters

Semi major axis (a)	6,378,145 metres
Inverse Flattening (1/f)	298.25

The rectangular grid coordinate system used in conjunction with the Clarke 1858 spheroid was called the Australian National Grid (ANG) (NMCA 1976), but was also known as the Australian Transverse Mercator (ATM). "*Coordinates were quoted in yards and were derived from a Transverse Mercator projection of latitudes and longitudes determined in relation to the relevant State or local coordinate origin*" (NMCA 1986). A discussion of the development of this system can be found in Lines (1992).

Table 1-6: ANG Parameters

Central scale factor	1.0 exactly
False Easting	400,000 yards
False Northing	800,000 yards
Zone Width	5 degrees
Initial Central meridian (Zone one)	116 degrees east longitude

** The true origin for each zone of the ANG was the intersection of the central meridian and S34° latitude, with the false origin 800,000 yards further south.*

*(**Note**: this is equivalent to a false northing of 4,915,813.467 yards from the equator = 4,115,813.469 + 800,000 yards). In Tasmania, to prevent negative coordinates, a further 1,000,000 yards was added to the false northing (total 1,800,000 yards) (A.H.Q. 1942).*

Chapter 2 Reduction of Measured Distances to the Ellipsoid

Excel Spreadsheet – Calculation of Reduced Distance

Due to the effects of atmospheric refraction, the light waves or microwaves used by EDM follow a curved path. Before this curved wave path distance can be used for any geodetic computations, it should be reduced to the surface of the ellipsoid by the application of both physical and geometric corrections. Figure 2-1 illustrates the situation.

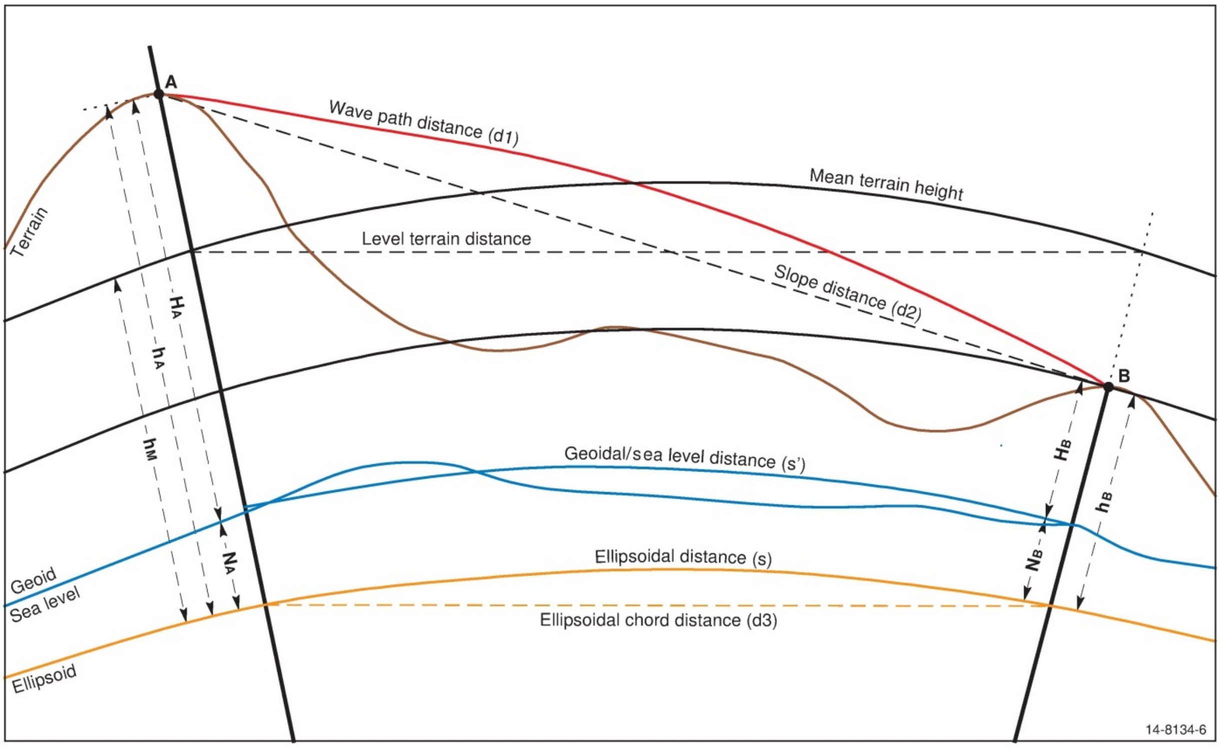


Figure 2-1: Reduction of distance to the Ellipsoid

The difference between the wave path length (d_1) and the wave path chord (d_2) is a function of the EDM equipment used and also of the meteorological conditions prevailing along the wave path at the time of measurement. This difference can often be ignored for distance measurements of up to 15 kilometres, using either light waves or microwaves. These physical corrections, which involve the application of certain velocity corrections to the measured wave path distance, are not discussed in this manual.

Combined Formula

The reduction of the wave path chord distance (d_2), to the ellipsoidal chord distance (d_3), can be given as a single rigorous formula (Clark 1966):

$$d_3 = \left[\left({d_2}^2 - (h_A - h_B)^2\right)/(1 + h_A/R_\alpha)(1 + h_B/R_\alpha)\right]^{1/2}$$

The ellipsoidal chord distance (d_3) is then easily reduced to the ellipsoidal distance:

$$s = d_3\left[1 + \left({d_3}^2/24{R_\alpha}^2 + 3{d_3}^4/640{R_\alpha}^4 + \ldots\right)\right]$$

where R_α is the radius of curvature in the azimuth of the line.

For a distance of 30 kilometres in the Australian region the chord-to-arc correction is 0.028 m. For a distance of 50 km, the correction reaches about 0.13 m and it is more than 1 m at 100 km. The second term in the chord-to-arc correction is less than 1 mm for lines up to 100 km, anywhere in Australia and usually can be ignored.

Separate Formulae

The combined formula above includes the slope and ellipsoid level corrections. The slope correction reduces the wave path chord (d_2) to a horizontal distance at the mean elevation of the terminals of the line and the ellipsoid level correction reduces the horizontal distance to the ellipsoid chord distance (d_3). The chord-to-arc correction is then applied to the ellipsoid chord distance, as with the combined formula, to give the ellipsoidal distance (s).

$$\text{Slope correction} = \left({d_2}^2 - \Delta h^2\right)^{1/2} - d_2$$

$$\text{Ellipsoidal correction} = (h_m/R_\alpha)\left({d_2}^2 - \Delta h^2\right)^{1/2}$$

$$\text{Chord to arc correction} = +\,{d_3}^3/24{R_\alpha}^2\{+\,3{d_3}^5/640{R_\alpha}^4 + \ldots\}$$

Heights in Distance Reduction

The formulae given in this chapter use ellipsoidal heights (*h*). If the geoid-ellipsoid separation (Chapter 9 – *N* value) is ignored and only the height above the geoid (*H* – the orthometric or AHD height) is used, an error of one part per million (ppm) will be introduced for every 6.5 m of *N* value (plus any error due to the change in *N* value along the line). As the *N* value in terms of GDA varies from -35 m in southwest Australia, to about 70 m in northern Queensland, errors from -5 to almost 11 ppm could be expected. Of course there are areas where the *N* value is small and the error would also be small.

Radius of Curvature

The radius of curvature is a function of latitude and for many applications the geometric mean radius ($R_{m,}$) (Figure 2-2), can be used rather than the radius in the azimuth of the line (R_α). However, there can be a large difference between the geometric mean radius and the radius in the azimuth of the line.

For high accuracy applications the radius of curvature in the azimuth of the line should be used.

$R_m = (\rho\nu)^{\frac{1}{2}}$ and

$$R_\alpha = (\rho\nu)/(\nu\cos^2\alpha + \rho\sin^2\alpha)$$

where:

$$\rho = a(1 - e^2)/(1 - e^2 sin^2\phi)^{3/2}$$

$$\upsilon = a/(1 - e^2 sin^2 \phi)^{1/2}$$

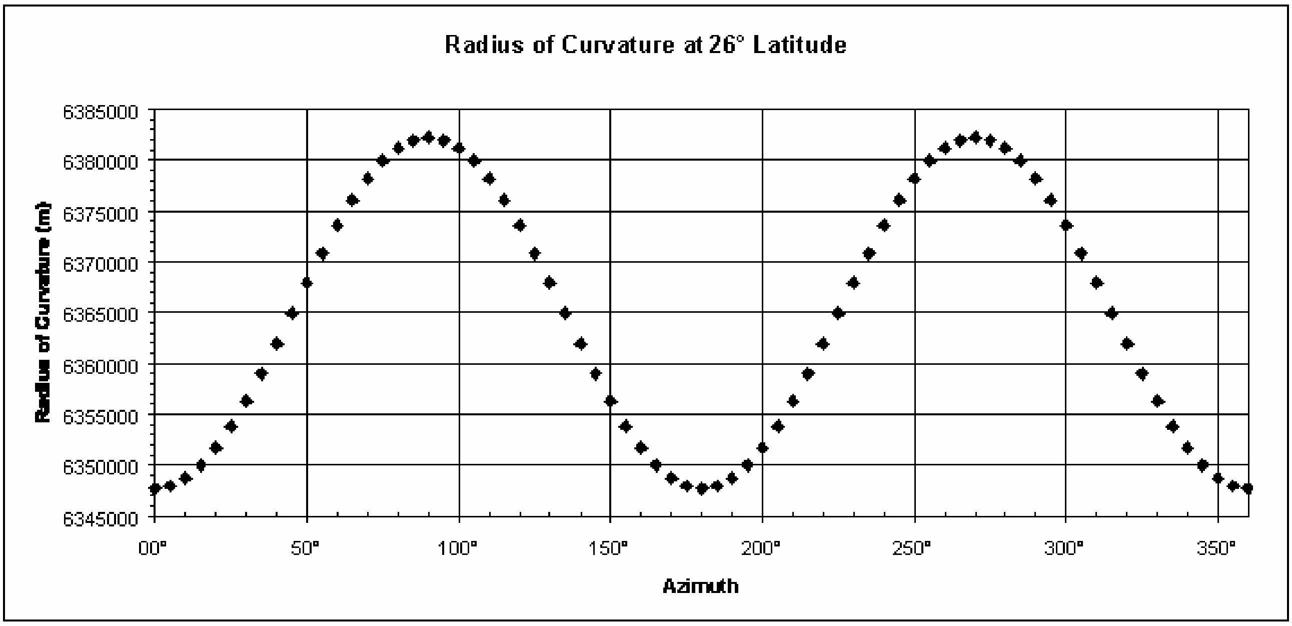


Figure 2-2: Radius of Curvature for Latitude 26°

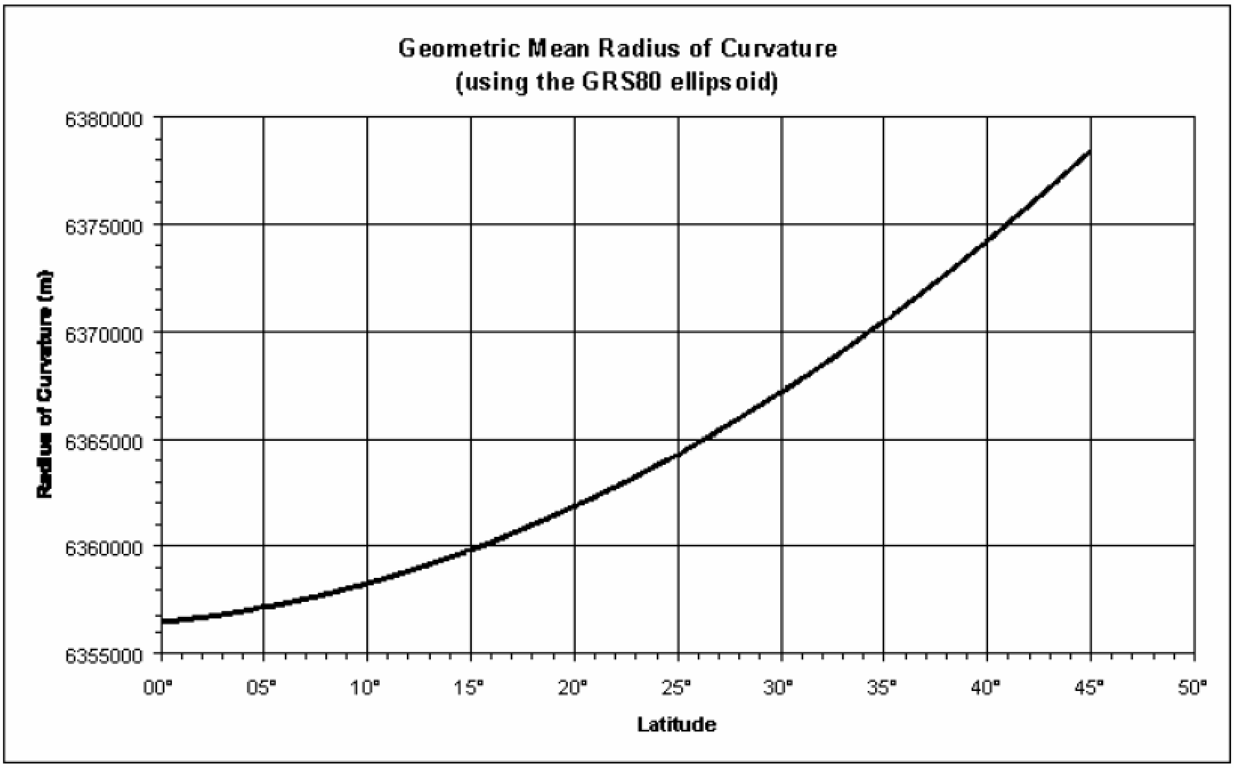


Figure 2-3: Radius of Curvature

Chapter 3 Reduction of Measured Directions to the Ellipsoid

Excel Spreadsheet – Calculation of Deflection & Laplace Corrections

When a total station is levelled to make an angular observation (direction or azimuth) it is levelled according to the plumbline at that point, i.e. the normal to the geoid. This is generally different from the normal to the ellipsoid at the same point. This difference is known as the deflection of the vertical. The correction for this deflection is generally small, but should be applied for the highest quality results. Deflections of the vertical can be computed from astronomic and geodetic coordinates at the same point, or they can be produced from a geoid model such as AUSGeoid.

A further correction can be made to account for the fact that the normals at each end of the line are not parallel (the skew normal correction). This too is a small correction and "*except in mountainous country, it can reasonably be ignored*". (Bomford 1980).

Because they are related to a particular ellipsoid, deflections of the vertical, like geoid ellipsoid separations, will be different for different datums. Within Australia, the maximum deflection in terms of the GDA is of the order of twenty seconds of arc, which could result in a correction to an observed direction or azimuth approaching half a second of arc.

The Laplace correction defines the relationship between an astronomically observed azimuth and a geodetic azimuth. It can be a significant correction, of the order of several seconds of arc, and should always be applied to an astronomic azimuth before computing coordinates.

The formulae for these corrections are often given using the astronomic convention, with east longitude negative. However, the formulae used here have been rearranged to use the geodetic conventions, as used elsewhere in this manual (east longitude positive).

Formulae

$$\begin{aligned}\text{Direction (reduced)} = \text{Direction (measured)} \\ &+ \text{Deflection correction} \\ &+ \text{Skew normal correction} \\ &+ \text{Laplace correction (Laplace for azimuth only)}\end{aligned}$$

$$\text{Deflection correction} = -\zeta \tan e$$

$$\text{where: } \zeta = \xi \sin \alpha - \eta \cos \alpha$$

If the elevation angle (e) is not known, an effective estimate can be obtained from:

$$\tan e = [(H_2 - H_1) - 0.067\, D^2]\,/1000D$$

$$\text{Skew normal correction} = e'^2 H_2 \cos^2 \Phi \sin(2\alpha)/2R$$

$$\text{Laplace correction} = (\lambda_A - \lambda_G) \sin \Phi$$

Sample Data

Kaputar to NM C 59 – GDA94	
Height of Kaputar (H_1)	1507.89
Height of NM C 59 (H_2)	217.058
Distance 1 to 2 (km)	58.120
Computed Elevation angle (e)	-1° 29' 43"
Geodetic Latitude Kaputar (Φ_G)	-30° 16' 24.4620"
Geodetic Longitude Kaputar (λ_G)	150° 09' 52.0945"
Observed Astronomic values	
Astro latitude Kaputar (Φ_A)	-30° 16' 25.580"
Astro longitude Kaputar (λ_A)	150° 09' 40.050"
Deflections Calculated from Astro	
Meridian component deflection (ξ)	-01.118"
Prime vertical component (η)	-10.402"
Deflections from AUSGeoid09	
Meridian component deflection (ξ)	-2.58"
Prime vertical component (η)	-10.74"
Elevation angle (e)	-01° 29' 43"
Corrections to Azimuth	
Astro Azimuth (α_A) (observed)	265° 25' 30.520"
+Deflection Correction (using AUSGeoid deflections)	00.063"
+Skew normal correction (using AUSGeoid deflections)	00.003"
+Laplace correction (using astro deflection)	-06.072"
=Geodetic Azimuth (α_G)	265° 25' 24.514"

Symbols

ξ the component of the deflection of the vertical in the meridian, in seconds of arc

= astronomic latitude – geodetic latitude

η the component of the deflection of the vertical in the prime vertical, in seconds of arc.

= (astronomic longitude – geodetic longitude) $\cos \Phi$

α Azimuth of the observed line – *A* (astronomic) *G* (geodetic)

e the elevation angle of the observed line (positive or negative)

R Radius of the earth in metres. For these small corrections, any reasonable estimate may be used.

H_2 Height of the reference station in metres.

H_1 Height of the observing station in metres.

D Distance between the observing and reference stations in kilometres.

Chapter 4 Computations on the Ellipsoid

Excel Spreadsheet – Vincenty's Formulae (Direct and Inverse)

There are a number of formulae available to calculate accurate geodetic positions, azimuths and distances on the ellipsoid (Bomford 1980). Vincenty's formulae (Vincenty 1975) may be used for lines ranging from a few cm to nearly 20,000 km, with millimetre accuracy. The formulae have been extensively tested for the Australian region, by comparison with results from other formulae (Rainsford 1955; Sodano 1965).

Vincenty's Inverse formulae

Given: latitude and longitude of two points (Φ_1, λ_1 *and* Φ_2, λ_2),

Calculate: the ellipsoidal distance (s) and forward and reverse azimuths between the points ($\alpha_{1-2}, \alpha_{2-1}$).

$$\tan U_1 = (1 - f)\tan \Phi_1$$

$$\tan U_2 = (1 - f)\tan \Phi_2$$

Starting with the approximation,

$$\lambda = \omega = \lambda_2 - \lambda_1$$

Iterate the following equations, until there is no significant change in σ:

$$\sin^2\sigma = (\cos U_2 \sin\lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda)^2$$

$$\cos\sigma = \sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos\lambda$$

$$\tan\sigma = \sin\sigma / \cos\sigma$$

$$\sin\alpha = \cos U_1 \cos U_2 \sin\lambda / \sin\sigma$$

$$\cos 2\sigma_m = \cos\sigma - (2\sin U_1 \sin U_2 / \cos^2\alpha)$$

$$C = (f/16)\cos^2\alpha\,[4 + f(4 - 3\cos^2\alpha)]$$

$$\lambda = \omega + (1 - C) f \sin\alpha\,\{\sigma + C\sin\sigma[\cos 2\sigma_m + C\cos\sigma\,(-1 + 2\cos^2 2\sigma_m)]\}$$

Then:

$$u^2 = cos^2\alpha\,(a^2 - b^2)/b^2$$

$$A = 1 + (u^2/16384)\,\{4096 + u^2[-768 + u^2(320 - 175u^2)]\}$$

$$B = (u^2/1024)\,\{256 + u^2[-128 + u^2(74 - 47u^2)]\}$$

$$\Delta\sigma = B\sin\sigma\,\{\cos 2\sigma_m + (B/4)\,[\cos\sigma(-1 + 2\cos^2 2\sigma_m) - (B/6)\cos 2\sigma_m(-3 + 4\sin^2\sigma)(-3 + 4\cos^2 2\sigma_m)]\}$$

$$s = bA(\sigma - \Delta\sigma)$$

$$\tan\alpha_{1-2} = (\cos U_2 \sin\lambda)/(\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda)$$

$$\tan\alpha_{2-1} = (\cos U_1 \sin\lambda)/(-\sin U_1 \cos U_2 + \cos U_1 \sin U_2 \cos\lambda)$$

Vincenty's Direct formulae

Given: latitude and longitude of a point (Φ_1, λ_1) and the geodetic azimuth (α_{1-2}) and ellipsoidal distance to a second point (s),

Calculate: the latitude and longitude of the second point (Φ_2, λ_2) and the reverse azimuth (α_{2-1}).

$$\tan U_1 = (1-f)\tan \Phi_1$$

$$\tan \sigma_1 = \tan U_1 / \cos \alpha_{1-2}$$

$$\sin \alpha = \cos U_1 \sin \alpha_{1-2}$$

$$u^2 = \cos^2\alpha\,(a^2 - b^2)/b^2$$

$$A = 1 + (u^2/16384)\,\{4096 + u^2[-768 + u^2(320 - 175u^2)]\}$$

$$B = (u^2/1024)\,\{256 + u^2[-128 + u^2(74 - 47u^2)]\}$$

Starting with the approximation:

$$\sigma = (s/bA)$$

Iterate the following three equations until there is no significant change in

$$2\sigma_m = 2\sigma_1 + \sigma$$

$$\Delta\sigma = B\sin\sigma\,\{\cos 2\sigma_m + (B/4)\,[\cos\sigma(-1 + 2\cos^2 2\sigma_m) - (B/6)\cos 2\sigma_m(-3 + 4\sin^2\sigma)(-3 + 4\cos^2 2\sigma_m)]\}$$

$$\sigma = (s/bA) + \Delta\sigma$$

Then:

$$\tan \Phi_2 = (\sin U_1 \cos\sigma + \cos U_1 \sin\sigma \cos\alpha_{1-2}) / \left\{(1-f)[\sin^2\alpha + (\sin U_1 \sin\sigma - \cos U_1 \cos\sigma\cos\alpha_{1-2})^2]^{\frac{1}{2}}\right\}$$

$$\tan \lambda = (\sin\sigma \sin\alpha_{1-2}) / (\cos U_1 \cos\sigma - \sin U_1 \sin\sigma \cos\alpha_{1-2})$$

$$C = (f/16)\cos^2\alpha[4 + f(4 - 3\cos^2\alpha)]$$

$$\omega = \lambda - (1-C)f\sin\alpha\{\sigma + C\sin\sigma[\cos 2\sigma_m + C\cos\sigma(-1 + 2cos^2 2\sigma_m)]\}$$

$$\lambda_2 = \lambda_1 + \omega$$

$$\tan \alpha_{2-1} = (\sin\alpha) / (-\sin U_1 \sin\sigma + \cos U_1 \cos\sigma\cos\alpha_{1-2})$$

Note:

• *"The inverse formulae may give no solution over a line between two nearly antipodal points. This will occur when λ is greater than π in absolute value."* (Vincenty 1975)

• In Vincenty (1975) L is used for the difference in longitude, however for consistency with other formulae in this Manual, ω is used here.

• Variables specific to Vincenty's formulae are shown below, others common throughout the manual are shown in the **Error! Reference source not found.**.

Sample Data

Flinders Peak	-37° 57' 03.72030"	144° 25' 29.52440"
Buninyong	-37° 39' 10.15610"	143° 55' 35.38390"
Ellipsoidal Distance	54,972.271 m	
Forward Azimuth	306° 52' 05.37"	
Reverse Azimuth	127° 10' 25.07"	

Symbols

α	Azimuth of the geodesic at the equator. (Forward$_{1\text{-}2}$, Reverse$_{2\text{-}1}$)
U	Reduced latitude
λ	Difference in longitude on an auxiliary sphere (λ_1 & λ_2 are the geodetic longitudes of points 1 & 2
σ	Angular distance on a sphere, from point 1 to point 2
σ_1	Angular distance on a sphere, from the equator to point
σ_2	Angular distance on a sphere, from the equator to point 2
σ_m	Angular distance on a sphere, from the equator to the midpoint of the line from point 1 to point 2
U, A, B, C	Internal variables

Chapter 5 Conversion between Ellipsoidal and Grid Coordinates

Excel Spreadsheet – Redfearn's Formulae

Redfearn's formulae were published in the "Empire Survey Review", No. 69, (1948). They may be used to convert between latitude & longitude and easting, northing & zone for a Transverse Mercator projection, such as the Map Grid of Australia (MGA). These formulae are accurate to better than 1 mm in any zone of the Map Grid of Australia and for the purposes of definition may be regarded as exact.

Preliminary Calculations

Meridian Distance

To evaluate Redfearn's formulae length of an arc of a meridian must be computed. This is given by

$$m = a(1-e^2)\int_{\Phi_1}^{\Phi_2}[1-(e^2 sin^2\Phi)]^{-3/2}d\Phi$$

where Φ_1 and Φ_2 are the latitudes of the starting and finishing points. When calculating the meridian distance from the equator, Φ_1 becomes zero. This formula may be evaluated by an iterative method (such as Simpson's rule) but it is more efficient to use a series expansion, as shown below.

$$m = a\{A_0\Phi - A_2\sin 2\Phi + A_4\sin 4\Phi - A_6\sin 6\Phi\}$$

where:

$$A_0 = 1-(e^2/4)-(3e^4/64)-(5e^6/256)$$

$$A_2 = (3/8)(e^2+e^4/4+15e^6/128)$$

$$A_4 = (15/256)(e^4+3e^6/4)$$

$$A_6 = 35e^6/3072$$

When the GRS80 ellipsoid parameters for the Map Grid of Australia are substituted, this formula for meridian distance reduces to the one shown below. However, to maintain flexibility when writing a computer program, the previous series expansion should be used.

$$m = 111132.952547\Phi \\ - 16038.50841\sin 2\Phi + 16.83220089\sin 4\Phi - 0.021800767\sin 6\Phi$$

where Φ in the first term is in degrees and 111132.952547 is the mean length of 1 degree of latitude in metres (G).

Foot-point Latitude

The foot-point latitude (Φ') is the latitude for which the meridian distance equals the true northing divided by the central scale factor ($m = N'/k_0$). This value can be calculated directly, once three other values are available.

$$n = (a-b)/(a+b) = f/(2-f)$$

$$G = a(1-n)(1-n^2)(1+(9/4)n^2+(225/64)n^4)(\pi/180)$$

$$\sigma = (m\pi)/(180G)$$

The foot point latitude (in radians) is then calculated by:

$$\begin{aligned}\Phi' = \sigma + \big((3n/2) - (27n^3/32)\big)\sin 2\sigma \\ + \big((21n^2/16) - (55n^4/32)\big)\sin 4\sigma + (151n^3/96)\sin 6\sigma \\ + (1097n^4/512)\sin 8\sigma\end{aligned}$$

Radius of Curvature

The radii of curvature for a given Latitude are also required in the evaluation of Redfearn's formulae.

$$\rho = a(1-e^2)/(1-e^2 sin^2\phi)^{3/2}$$

$$\nu = a/(1-e^2 sin^2\phi)^{1/2}$$

$$\Psi = \nu/\rho$$

Geographical to Grid

$$t = \tan\Phi$$

$$\omega = \lambda - \lambda_0$$

$$E' = (K_0\nu\,\omega\cos\Phi)\,\{1 + Term1 + Term2 + Term3\}$$

$$Term1 = (\omega^2/6)\cos^2\Phi\,(\Psi - t^2)$$

$$Term2 = (\omega^4/120)\cos^4\Phi\,[4\Psi^3(1-6t^2) + \Psi^2(1+8t^2) - \Psi 2t^2 + t^4]$$

$$Term3 = (\omega^6/5040)\cos^6\Phi\,(61 - 479t^2 + 179t^4 - t^6)$$

$$E = E' + False\ Easting$$

$$N' = K_0\{m + Term1 + Term2 + Term3 + Term4\}$$

$$Term1 = (\omega^2/2)\,\nu\sin\Phi\cos\Phi$$

$$Term2 = (\omega^4/24)\,\nu\sin\Phi\cos^3\Phi\,(4\Psi^2 + \Psi - t^2)$$

$$\begin{aligned}Term3 = (\omega^6/720)\,\nu\sin\Phi\cos^5\Phi\,[8\Psi^4(11-24t^2) - 28\Psi^3(1-6t^2) + \Psi^2(1-32t^2) \\ - \Psi(2t^2) + t^4]\end{aligned}$$

$$Term4 = (\omega^8/40320)\,\nu\sin\Phi\cos^7\Phi\,(1385 - 3111t^2 + 543t^4 - t^6)$$

$$N = N' + False\ Northing$$

Grid Convergence

$$\gamma = Term1 + Term2 + Term3 + Term4$$

where:

$$Term1 = -\omega\sin\Phi$$

$$Term2 = -(\omega^3/3)\sin\Phi\cos^2\Phi\,(2\Psi^2 - \Psi)$$

$$\begin{aligned}Term3 = -(\omega^5/15)\sin\Phi\cos^4\Phi\,[\Psi^4(11-24t^2) - \Psi^3(11-36t^2) + 2\Psi^2(1-7t^2) \\ + \Psi t^2]\end{aligned}$$

$$Term4 = -(\omega^7/315)\sin\Phi\cos^6\Phi\,(17 - 26t^2 + 2t^4)$$

Point Scale Factor

$$k = k_0 + k_0 Term1 + k_0 Term2 + k_0 Term3$$

$$Term1 = (\omega^2/2)\Psi \cos^2 \Phi$$

$$Term2 = (\omega^4/24) \cos^4 \Phi \, [4\Psi^3(1 - 6t^2) + \Psi^2(1 + 24t^2) - 4\Psi t^2]$$

$$Term3 = (\omega^6/720) \cos^6 \Phi \, (61 - 148t^2 + 16t^4)$$

Grid to Geographical

In the following formulae t, ρ, ν and Ψ are all evaluated for the foot point latitude.

$$E' = E - False\ Easting$$

$$x = E'/(K_0\nu')$$

$$\Phi = \Phi' - Term1 + Term2 - Term3 + Term4$$

$$Term1 = (t'/(K_0\rho'))(xE'/2)$$

$$Term2 = (t'/(K_0\rho'))(E'x^3/24)\left[-4\Psi'^2 + 9\Psi'(1 - t'^2) + 12t'^2\right]$$

$$Term3 = (t'/(K_0\rho'))(E'x^5/720)[8\Psi'^4(11 - 24t'^2) - 12\Psi'^3(21 - 71t'^2) \\ + 15\Psi'^2(15 - 98t'^2 + 15t'^4) + 180\Psi'(5t'^2 - 3t'^4) + 360t'^4]$$

$$Term4 = (t'/(K_0\rho'))(E'x^7/40320)(1385 + 3633t'^2 + 4095t'^4 + 1575t'^6)$$

$$\omega = Term1 - Term2 + Term3 - Term4$$

$$Term1 = x \sec \Phi'$$

$$Term2 = (x^3/6) \sec \Phi' \, (\Psi' + 2t'^2)$$

$$Term3 = (x^5/120) \sec \Phi' \left[-4\Psi'^3(1 - 6t'^2) + \Psi'^2(9 - 68t'^2) + 72\Psi' t'^2 + 24t'^4\right]$$

$$Term4 = (x^7/5040) \sec \Phi' \, (61 + 662t'^2 + 1320t'^4 + 720t'^6)$$

$$\lambda = \lambda_0 + \omega$$

Grid Convergence

$$x = E'/k_0\nu'$$

$$t' = \tan \Phi'$$

$$\gamma = Term1 + Term2 + Term3 + Term4$$

$$Term1 = -t'x$$

$$Term2 = (t'x^3/3)(-2\Psi'^2 + 3\Psi' + t'^2)$$

$$Term3 = (-t'x^5/15)\left[\Psi'^4(11 - 24t'^2) - 3\Psi'^3(8 - 23t'^2) + 5\Psi'^2(3 - 14t'^2) + 30\Psi' t'^2 \\ + 3t'^4\right]$$

$$Term4 = (t'x^7/315)(17 + 77t'^2 + 105t'^4 + 45t'^6)$$

Point Scale

$$x = \left(E'^2/k_0{}^2\rho'\nu'\right)$$

$$K = k_0 + k_0 Term1 + k_0 Term2 + k_0 Term3$$

$$Term1 = x/2$$

$$Term2 = (x^2/24)[4\Psi'(1 - 6t'^2) - 3(1 - 16t'^2) - 24t'^2/\Psi']$$

$$Term3 = x^3/720$$

Sample Data

Flinders Peak

MGA94 (zone 55)	E 273 741.297	N 5 796 489.777
GDA94	-37° 57' 03.7203"	144° 25' 29.5244"
Convergence	-01° 35' 03.65"	
Point scale factor	1.000 230 56	

Chapter 6 Grid Calculations

Excel Spreadsheet – Grid Calculations

Coordinates and the relationships between them are rigorously calculated using ellipsoidal formulae. These formulae produce geodetic coordinates (latitude and longitude), azimuths and ellipsoidal distances and are well within the scope of modern personal computers.

Redfearn's formulae can then be used to rigorously produce grid coordinates (easting, northing & zone), together with the point scale factor and convergence, from the geodetic coordinates; these can then be used to compute grid distances and grid bearings. Alternatively, the formulae given in this section can be used to compute grid coordinates, grid distances and grid bearings.

Grid Bearing and Ellipsoidal Distance from MGA94 coordinates

The following formulae provide the only direct method to obtain grid bearings and ellipsoidal distance from MGA94 coordinates.

$$\tan\theta_1 = (E_2' - E_1')/(N_2 - N_1)$$

or

$$\cot\theta_1 = (N_2 - N_1)/(E_2' - E_1')$$

$$L = (E_2' - E_1')/\sin\theta_1$$

$$= (N_2 - N_1)/\cos\theta_1$$

$$K = k_0\{1 + [({E_1'}^2 + E_1'E_2' + {E_2'}^2)/6{r_m}^2][1 + ({E_1'}^2 + E_1'E_2' + {E_2'}^2)/36{r_m}^2]\}$$

$$s = L/K$$

$$\delta_1{}'' = -(N_2 - N_1)(E_2' + 2E_1')[1 - (E_2' + 2E_1')^2/27{r_m}^2]/6{r_m}^2\{radians\}$$

$$\delta_1{}'' = 206264.8062\ \delta_1\{seconds\}$$

$$\delta_2{}'' = (N_2 - N_1)(2E_2' + E_1')[1 - (2E_2' + E_1')^2/27{r_m}^2]/6{r_m}^2\{radians\}$$

$$\delta_2{}'' = 206264.8062\ \delta_2\{seconds\}$$

$$\beta_1 = \theta_1 - \delta_1$$

$$\beta_2 = \theta_1 \pm 180° - \delta_2$$

The mean radius of curvature can be calculated as shown below, using an approximate value for the mean latitude (Φ_m'). The approximate mean latitude can be calculated in two steps, with an accuracy of about two minutes of arc, using the formulae shown below. This approximation is derived from the formulae for meridian distance used with Redfearn's formulae and the constants shown are the values aA_1 and aA_2, computed for GDA.

$$N' = N - False\ Northing$$

$$N_m' = (N_1' + N_2')/2$$

$$\Phi_m'(1st\ approx) = (N_m'/k_0)/111132.952$$

$$\Phi_m'(2nd\ approx) = ((N_m'/k_0) + 16038.508\ \sin 2\Phi_m')/111132.952$$

$$\rho_m = a(1 - e^2)/(1 - e^2\sin^2\Phi_m')^{3/2}$$

$$\nu_m = a/(1 - e^2\sin^2\Phi_m')^{1/2}$$

$$r_m{}^2 = \rho_m \nu_m k_o{}^2$$

MGA94 Coordinates from Grid bearing and Ellipsoidal Distance

This computation is commonly used when the coordinates of one station are known and the grid bearing and ellipsoidal distance from this station to an adjacent station have been determined. The bearing and distance are applied to the coordinates of the known station to derive the coordinates of the unknown station and the reverse grid bearing. The formulae shown are accurate to 0.02" and 0.1 ppm over any 100 kilometre line in an MGA zone. For lower order surveys:

- the underlined terms are often omitted
- the latitude function $1/6r^2$ becomes a constant and
- the formulae for K and δ are replaced by simplified versions

Formulae

First calculate approximate coordinates for the unknown station:

- $E_1' = E_1 - 500\,000$
- $E_2' \approx E'_1 + k_1 s \sin\beta_1$
- $N_2 - N_1 \approx k_1 s \cos\beta_1$

If not already known the point scale factor (k_1) may be approximated by:

$$k_1 \approx 0.9996 + 1.23E'^2 10^{-14}$$

$$K = k_0\left\{1 + \left[\left(E_1'^2 + E_1'E_2' + E_2'^2\right)/6r_m{}^2\right]\underline{\left[1 + \left(E_1'^2 + E_1'E_2' + E_2'^2\right)/36r_m{}^2\right]}\right\}$$

$$L = sK$$

$$\sin\delta_1 = -(N_2 - N_1)(E_2' + 2E_1')\left[\underline{1 - (E_2' + 2E_1')^2/27r_m{}^2}\right]/6r_m{}^2$$

$$\theta = \beta_1 + \delta_1$$

$$\sin\delta_2 = (N_2 - N_1)(2E_2' + E_1')\left[\underline{1 - (2E_2' + E_1')^2/27r_m{}^2}\right]/6r_m{}^2$$

$$\beta_2 = \theta \pm 180° - \delta_2$$

$$\Delta E = L\sin\theta$$

$$\Delta N = L\cos\theta$$

$$E_2 = E_1 + \Delta E$$

$$N_2 = N_1 + \Delta N$$

The mean radius of curvature can be calculated as shown below, using an approximate value for the mean latitude (Φ_m'). The approximate mean latitude can be calculated in two steps, with an accuracy of about two minutes of arc, using the formulae shown below. This approximation is derived from the formulae for meridian distance used with Redfearn's formulae and the constants shown are the values aA_0 and aA_2, computed for GDA.

$$N' = N - False\ Northing$$

$$N_m' = (N_1' + N_2')/2$$

$$\Phi'_m(1st\ approx) = (N'_m/k_0)/111132.952$$

$$\Phi'_m(2nd\ approx) = \left((N'_m/k_0) + 16038.508 \sin 2\Phi'_m\right)/111132.952$$

$$\rho_m = a(1-e^2)/(1-e^2 \sin^2 \Phi'_m)^{3/2}$$

$$\nu_m = a/(1-e^2 \sin^2 \Phi'_m)^{1/2}$$

$${r_m}^2 = \rho_m \nu_m {k_0}^2$$

Zone to Zone Transformations

If a point lies within 0.5° of a zone boundary, it is possible to compute the grid coordinate of the point in terms of the adjacent zone. This can be done by:

1. converting the known grid coordinates to latitude and longitude using Redfearn's formulae, and then converting back to grid coordinates in terms of the adjacent zone, or

2. using the formulae shown below (Jordan and Eggert 1941; Grossmann 1964). These formulae have an accuracy of 10 mm anywhere within 0.5° of a zone boundary.

Formulae

$$\tan J_1 = [{\omega_z}^2 \cos^2 \Phi_z (1 + 31 \tan^2 \Phi_z) - 6(1 + e'^2 \cos^2 \Phi_z)]/[18\omega_z \sin \Phi_z (1 + e'^2 \cos^2 \Phi_z)]$$

$$H_1 = -3{\omega_z}^2 \sin \Phi_z \cos \Phi_z /(\rho_z \cos J_1)$$

$$E_2 = 500\,000 - E'_z + (E'_1 - E'_z) \cos 2\gamma_z - (N_1 - N_z) \sin 2\gamma_z + H_1 L^2 \sin(2\theta_z + J_1)$$

$$N_2 = N_z + (N_1 - N_z) \cos 2\gamma_z + (E'_1 - E'_z) \sin 2\gamma_z + H_1 L^2 \cos(2\theta_z + J_1)$$

where:

Z	is a point on the zone boundary,
E_1, N_1	are the known coordinates of the point to be transformed,
E_2, N_2	are the coordinates of the point in terms of the adjacent zone,
θ_z	is the plane bearing from Z to the point to be transformed.

Traverse Computation with Grid Coordinates, using Arc-to-Chord Corrections and Line Scale Factors

With the power of modern computers, traverses can be rigorously computed on the ellipsoid, using formulae such as those shown in Chapter 4. The geographic results from these computations can then be rigorously converted to grid coordinates using Redfearn's formulae. However if necessary, the computation can be varied to suit the requirements of the job:

• the arc-to-chord corrections and line scale factors can be ignored and the traverse computed using the formulae of plane trigonometry;

• if good quality maps showing the MGA94 Grid are available, traverse stations may be plotted by inspection and the approximate coordinates scaled with sufficient precision to enable computation of the arc-to-chord corrections and line scale factors;

• the arc-to-chord corrections and line scale factors can be computed precisely, and the method becomes first order anywhere in a MGA94 grid zone.

The precision obtained should be closely balanced against the labour involved, though with modern Personal Computers and available software, the difference between a rigorous and approximate calculation is trivial. Prior to precise computation, approximate coordinates and bearings may be carried through the traverse, using uncorrected field measurements, to ensure that the observations are free of gross errors. A diagram of the traverse, approximately to scale, is often useful.

Basic Outline

There are many ways of arranging the computation. Essentially, the work is split into stages:

1. Approximate Eastings and Northings are computed from observed angles and distances;

2. Arc-to-chord corrections and line scale factors are computed from the approximate coordinates and applied to the observations to give plane angles and plane distances;

3. Precise coordinates are computed by plane trigonometry;

4. Misclosure in grid bearing and position is analysed and the traverse or figure adjusted as required.

For precise computation, each line is rigorously computed before the next line is calculated, so that errors in the approximate coordinates do not accumulate. True Eastings (E') and differences in northing (ΔN) are the quantities carried through the computation. Sign conventions may be disregarded and signs determined by inspection of a traverse diagram.

Formulae and Symbols

If the underlined terms shown in the preceding sections of this chapter are omitted, the errors for a 100 kilometre line running north and south on a zone boundary do not exceed 0.08" in bearing and 0.25 ppm in distance. For traverses of lower order, simplified formulae can be used. For short lines near a central meridian it may be possible to omit the arc-to-chord corrections and line scale factors and compute the traverse with observed angles and distances, using the formulae of plane trigonometry.

If the symbol δ_{21} is used for the arc-to-chord correction at station 2 to station 1 and δ_{23} for the correction at station 2 to station 3, and the angles are measured clockwise from station 1 to station 3, then the angle P_2 (plane) at station 2 is obtained from the angle 0_2 (observed) by:

$$P_2 = 0_2 + \delta_{23} - \delta_{21}$$

where angles are measured clockwise only.

Computations of ARC-TO-CHORD Corrections and scale factors

Although there are several ways of arranging the computation, the following is recommended:

1. Compute the grid bearing to the "forward" station by applying the observed horizontal angle at the "occupied" station to the known grid bearing of the "rear" station;

2. Compute the point scale factor at the "occupied" station and multiply the ellipsoidal distance to the "forward" station by this factor;

3. Using the distance obtained and the forward grid bearing, compute approximate coordinates of the "forward" station by plane trigonometry;

4. Using the coordinates of the "occupied" station and the approximate coordinates of the "forward" station, compute the arc-to-chord correction at the "occupied" station and the line scale factor. If the line crosses the central meridian, $(E1, E2)$ is negative;

5. Add the arc-to-chord correction to the forward grid bearing to obtain the plane bearing and multiply the spheroidal distance by the line scale factor to obtain the plane distance;

6. Using the plane bearing and plane distance, compute the coordinates of the "forward" station by plane trigonometry;

7. Compute the arc-to-chord correction from the new station to the previously occupied station and add this to the plane bearing reversed by 180° to obtain the reverse grid bearing from the new station.

The above process is repeated for each new line of a traverse with the reverse grid bearing of the previous line becoming the known grid bearing to the rear station.

Sample Data

	Flinders Peak	**Buninyong**
MGA94 (zone 55)	E 273 741.297	E 228 854.052
	N 5 796 489.777	N 5 828 259.038
Ellipsoidal Distance (m)	54,972.271	
Plane Distance (m)	54,992.279	
Grid Bearing	305° 17' 01.72"	125° 17' 41.86"
Arc to chord	+19.47"	-20.67"
Line scale factor	1.000 363 97	

Chapter 7 Transformation of Coordinates

The coordinates of a point will change depending on which datum the coordinates are referred to. To change a coordinate from one datum to another, a mathematical process known as transformation is used. This may be done in two or three dimensions and requires a number of points with positions known in terms of both datums ('common' points). The accuracy of the transformation depends on the method chosen and the accuracy, number and distribution of the 'common points'.

For transforming AGD66 or AGD84 coordinates to GDA the grid transformation process is the most accurate. For the sake of consistency it is recommended for all transformations in Australia. However, it is recognised that there are different user requirements, so less accurate transformation methods are also provided. As the different methods will give different results, metadata should be maintained, giving the accuracy and method used to obtain the transformed positions.

The transformation parameters supplied in this manual are between AGD and GDA94 and supersede all previous parameters, including those between AGD and WGS84, as GDA94 is the same as WGS84 for most practical applications. Transformation from ITRF to GDA is not covered in detail in this manual, but is discussed in Chapter 1, with a link to more detailed information. Software developed to support transformation from ITRF to GDA can be downloaded from the GDA Technical Manual Web page. In particular the RapidMap prepared downloadable.

High Accuracy Transformation (Grid Transformation)

Excel Spreadsheet – Test Data for Grid Transformation

National Transformation Grids for AGD66 and AGD84 are available from the GDA Technical Manual web site.

Ideally, the transformation process should be:

- *"Simple to apply*
- *computationally efficient*
- *unique in terms of the solution it provides*
- *rigorous*

The first two criteria are necessitated by the large volumes of data that will have to be transformed. The second two are based on the premise that the transformation process must not compromise the quality or topology of the original data. In this regard it is argued that, with careful development it is possible to improve data accuracy by incorporating a distortion model in the transformation process." (Collier, Argeseanu et al. 1998).

In 1997, ICSM adopted an approach for Australia that fitted the above criteria. This method is the same as that adopted in Canada, in that it uses files of coordinate shifts that compensate for distortions in the original data, as well as transforming between datums. The complex mathematical processing, based on many common points, is done prior to the production of the files of coordinate shifts (Collier, Argeseanu et al. 1998) and the user only has to perform a simple interpolation to obtain the required shifts, followed by a simple addition to perform the transformation. The files of coordinate shifts are provided in the Canadian format known as National Transformation version 2 (NTv2). The Australian NTv2 transformation files are provided in the binary format, but software provided by ICSM jurisdictions can readily convert them to ASCII format. Although there was some minor initial confusion with the original Australian-produced binary files, both the ASCII and binary formats now conform to the Canadian format that is used in many GIS packages. An in-depth explanation of the format can be found in Appendix C of the "GDAit" User Guide and the GDAit Software Documentation available from https://www.propertyandlandtitles.vic.gov.au/surveying/geodetic-survey/geocentric-datum-of-australia.

Interpolation software

Initially, each State and Territory produced a transformation grid file for its area and NSW and Victoria combined theirs into a single grid (SEA). These transformation grid files transformed from either AGD66 or AGD84, depending on which version of AGD was previously adopted by that jurisdiction. Several States also produced software to interpolate and apply the transformation shifts, either interactively or from a file of coordinates, using any grid file in NTv2 format. Victoria produced (GDAit), Queensland (GDAy) and NSW (Datumtran and GEOD).

National Transformation Grids

Two national transformation grid files are now available to replace the previous State & Territory grid files (Collier and Steed 2001).

1. A complete national coverage from AGD66 to GDA94. This coverage was generated using the latest algorithms with data from the previous AGD66 State & Territory grid files and AGD66 & GDA94 data from the National Geodetic Data Base. In NSW and Victoria the on-shore and close coastal areas of the previous combined State grid have been included in the national grid, but elsewhere there may be differences. These differences are generally small but may be larger near the State borders and in areas where there was little or no common data (e.g. offshore). The AGD66 national file also covers the offshore areas out to the Exclusive Economic Zone (EEZ). Although still in NTv2 format, a simple conformal (7-parameter) transformation was used to generate the shifts in these offshore areas (See Figure 7-1).

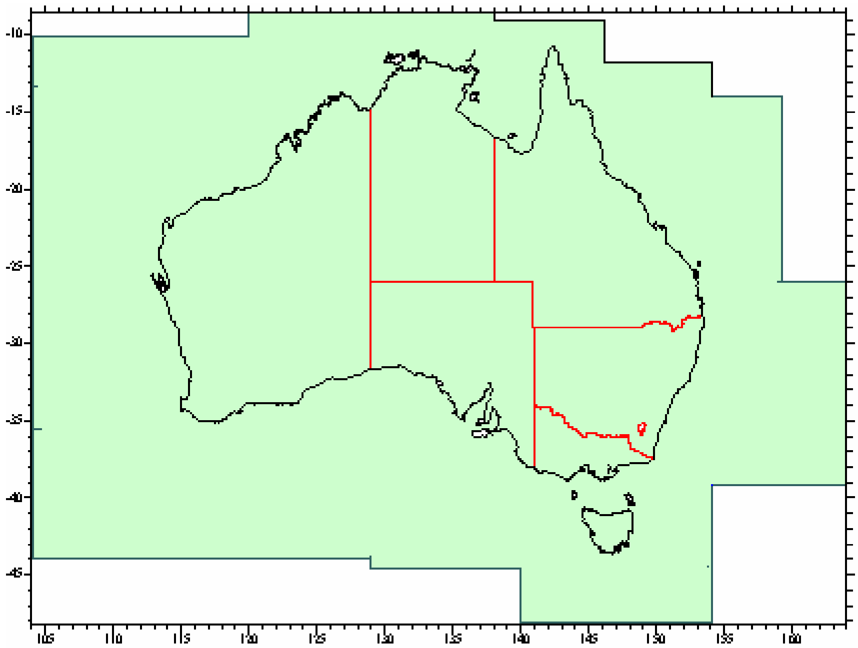


Figure 7-1 AGD66 to GDA94 Transformation Grid Coverage

2. A coverage from AGD84 to GDA94 for the States that previously adopted AGD84 (Queensland, South Australia and Western Australia). This coverage was produced by merging the existing Queensland, South Australian and West Australian transformation files and differs slightly from the previous State files only near the merged borders (see Figure 7-2).

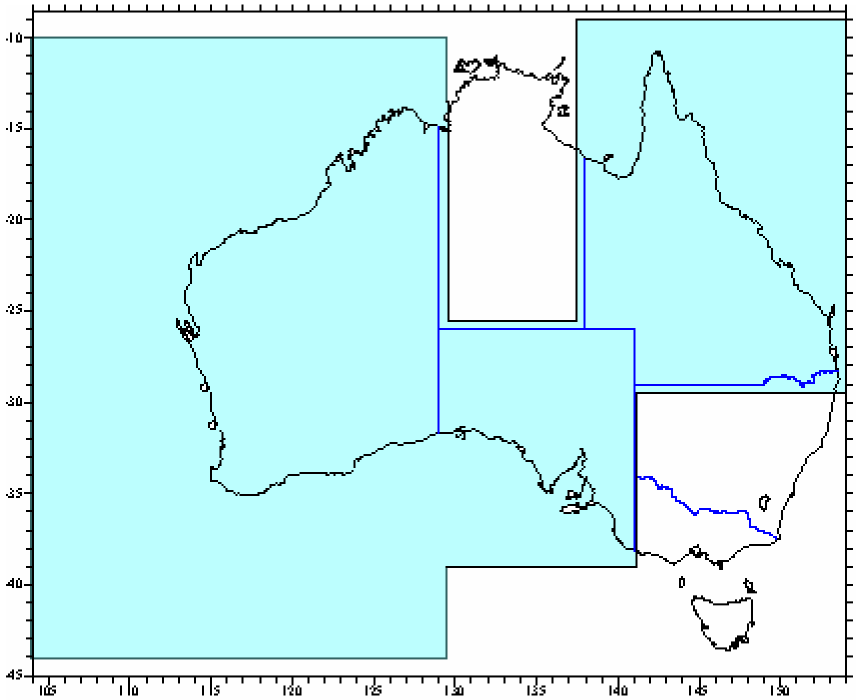


Figure 7-2 AGD 84 to GDA94 Transformation

For mathematical convenience and to suit the rectangular convention of the NTv2 format, the national grids extend outside the Australian EEZ in some places, but these extents do not infer any rights, nor do they imply the use of AGD or GDA94 coordinates in these areas.

For the convenience of those working only in a local area, software is also available to extract user defined areas from the national grid files from the ICSM GDA webpage.

To assist in the testing of transformation systems using these national grid files, a spreadsheet is available containing sample input and output for both the AGD66 & AGD84 grids on the ICSM GDA webpage.

Medium Accuracy Transformation

3-Dimensional Similarity Transformation

Excel Spreadsheet – Cartesian to Geodetic & 7-Parameter Transformation

Provided the rotation angles are small (a few seconds), the relationship between two consistent, three dimensional coordinate systems can be completely defined by a seven parameter similarity transformation (three origin shifts, three rotations and a scale change) (Harvey 1986).

The transformation is a relatively simple mathematical process, but because this technique is in terms of Earth-centred Cartesian coordinates (X Y Z), the points to be transformed must be converted to this coordinate type. This means that ellipsoidal heights are used on input and are produced on output; however, provided the ellipsoidal height entered is a reasonable estimate (within a few hundred metres) there will be negligible effect on the transformed horizontal position (millimetres).

National parameters to convert between AGD84 and GDA94 have been developed and have an estimated accuracy of about 1 m. Because of the inconsistent nature of the AGD66 coordinate set, it is not possible to compute a set of national AGD66/GDA94 parameters with acceptable accuracy, but they can be computed for local regions. Some authorities have computed regional AGD66/GDA94 parameters (see Regional Transformation Parameters from AGD66 to GDA94).

National parameters have been computed to transform between AGD84 and GDA94 using the similarity method. These parameters were computed from 327 points across Australia, which had both AGD84 and GDA94 coordinates, well determined AHD heights (by spirit levelling), and which were GPS points in the national GDA94 adjustment. The resulting parameters are shown in Table 7-1.

Note: *These parameters can be used for projects of medium accuracy (of the order 1 m). More accurate methods must be used for projects requiring greater accuracy. Although this method transforms the height, direct transformation of the height using the geoid-ellipsoid separation is easier and generally more accurate.*

Parameters

Table 7-1: National parameters – AGD84 and AGD66 to GDA94

Parameter	AGD84	AGD66
$\boldsymbol{DX}\ (\boldsymbol{m})$	-117.763	-117.808
$\boldsymbol{DY}\ (\boldsymbol{m})$	-51.510	-51.536
$\boldsymbol{DZ}\ (\boldsymbol{m})$	139.061	137.784
$\boldsymbol{R_X}(\boldsymbol{secs})$	-0.292	-0.303
$\boldsymbol{R_Y}(\boldsymbol{secs})$	-0.443	-0.446
$\boldsymbol{R_Z}\ (\boldsymbol{secs})$	-0.277	-0.234
$\boldsymbol{Sc}\ (\boldsymbol{ppm})$	-0.191	-0.290

The AGD84 parameters were tested using points additional to the initial 327, which had both AGD84 and GDA94 coordinates. A summary of these tests is shown in Table 7-2. The AGD66 parameters were developed as first step in the production of the national transformation grid and used 9,761 common points.

Table 7-2: AGD ↔ GDA94 parameters – residuals from 1571 points (lat/long) and 65 points (ellip. ht.)

	Average (m)	Std. Dev. (m)	Max (m)	Min (m)
Latitude	-0.10	0.38	1.03	-1.48
Longitude	-0.08	0.38	1.14	-2.50
Ellip. Ht.	0.14	0.37	0.95	-0.66

Formulae

Once the positions have been converted to Earth-centred Cartesian coordinates, the similarity transformation is performed by a simple matrix operation:

$$\begin{matrix} x' \\ y' \\ z' \end{matrix} = \begin{vmatrix} \Delta x \\ \Delta y \\ \Delta z \end{vmatrix} + (1 + Sc * 10^{-6})R \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

Where R is the combined matrix of rotations about the X, Y and Z axes, in that order, i.e.

$$R = R_x R_y R_z$$

In its full form this combined rotation matrix is:

$$\begin{vmatrix} \cos R_y \cos R_z & \cos R_y \sin R_z & -\sin R_y \\ \sin R_x \sin R_y \cos R_z - \cos R_x \sin R_z & \sin R_x \sin R_y \sin R_z + \cos R_x \cos R_z & \sin R_x \cos R_y \\ \cos R_x \sin R_y \cos R_z + \sin R_x \sin R_z & \cos R_x \sin R_y \sin R_z - \sin R_x \cos R_z & \cos R_x \cos R_y \end{vmatrix}$$

But for small rotations (a few seconds) it is closely approximated by the matrix below (where the rotations are in radians):

$$\begin{vmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{vmatrix}$$

Warning

There are two different ways of applying the sign conventions for the rotations. In both cases the sign convention is the same (a positive rotation is an anti-clockwise rotation, when viewed along the positive axis towards the origin) but:

1. The International Earth Rotation Service (IERS) assumes the rotations to be of the position around the coordinate axes, while

2. The method historically used in Australia assumes the rotations to be of the coordinate axes.

The only difference in the formula is a change in the signs of the angles in the rotation matrix. If the sign of the rotation parameters and the formulae used are consistent the correct results will be obtained. The only way to be absolutely sure which method or parameters are required is to test them using a known input and output for a set of parameters as shown in Table 7-3. If necessary the situation can be rectified by simply changing the sign of the rotation parameters.

Table 7-3: Sample input and output, using the national AGD84 Similarity parameters

	AGD84	**GDA94**
Latitude	S 37° 39' 15.5647"	S 37° 39' 10.1598"
Longitude	E 143° 55' 30.5501"	E 143° 55' 35.3730"
Ellipsoidal height	749.671 m	737.574 m

Table 7-4: Sample input and output, using the national AGD66 Similarity parameters

	AGD66	GDA94
Latitude	S 37° 39' 15.5571"	S 37° 39' 10.1757"
Longitude	E 143° 55' 30.6330"	E 143° 55' 35.4093"
Ellipsoidal height	749.671 m	737.739 m

Conversion between Geographical and Cartesian Coordinates

To convert between Geographical coordinates (latitude, longitude and ellipsoidal height) and three dimensional, Earth-centred Cartesian coordinates (X, Y, Z), the formulae given below are used.

It is essential that the appropriate reference ellipsoid is used and also to note that ellipsoidal heights must be used on input and are produced on output.

Formulae

Geographical to Cartesian	**Cartesian to Geographical**
$X = (\nu + h)\cos\Phi\cos\lambda$ $Y = (\nu + h)\cos\Phi\sin\lambda$ $Z = \{(1 - e^2)\nu + h\}\sin\Phi$	$\tan\lambda = Y/X$ $\tan\Phi = (Z(1-f) + e^2 a\sin^3 u)/\big((1-f)(p - e^2 a\cos^3 u)\big)$ $h = p\cos\Phi + Z\sin\Phi - a(1 - e^2\sin^2\Phi)^{1/2}$
Where: $\nu = a/\{(1 - e^2\sin^2\Phi)^{1/2}\}$ $e^2 = 2f - f^2$ $h = N + H$	Where: $p = (X^2 + Y^2)^{1/2}$ $\tan u = (Z/p)[(1-f) + (e^2\,a/r)]$ $r = (p^2 + Z^2)^{1/2}$

Example using GDA94 (GRS80 ellipsoid)

Latitude	-37° 39' 10.1598"	-4087095.384	**X**
Longitude	143° 55' 35.3730"	2977467.494	**Y**
Ellipsoidal height	737.574 m	-3875457.340	**Z**

Regional Transformation Parameters from AGD66 to GDA94

Although it is possible to compute national similarity transformation parameters between AGD84 & GDA94, AGD66/GDA94 similarity transformation parameters can only be accurately computed for smaller areas where AGD66 is more consistent. This was done as a first step in the development of the jurisdiction transformation grids and where the more accurate methods are not appropriate, these parameters may be used.

The parameters shown are only valid for transformation between AGD66 and GDA94 for the area indicated. They have an accuracy of only about 1 metre and the transformation grid method is preferred if at all possible.

Table 7-5: Regional Similarity transformation parameters – AGD66 to AGD94

Parameter	A.C.T	Tasmania	Victoria NSW	Northern Territory
DX (m)	-129.193	-120.271	-119.353	-124.133
DY (m)	-41.212	-64.543	-48.301	-42.003
DZ (m)	130.730	161.632	139.484	137.400
RX (secs)	-0.246	-0.217	-0.415	0.008
RY (secs)	-0.374	0.067	-0.260	-0.557
RZ (secs)	-0.329	0.129	-0.437	-0.178
Sc (ppm)	-2.955	2.499	-0.613	-1.854

Table 7-6: Sample input and output, using A.C.T. Similarity parameters

	AGD66	GDA94
Latitude	S 35° 18' 18.0000"	S 35° 18' 12.3911"
Longitude	E 149° 08' 18.0000"	E 149° 08' 22.3382"
Ellipsoidal height	600.000 m	601.632 m

Table 7-7: Sample input and output, using Tasmanian Similarity parameters

	AGD66	GDA94
Latitude	S 42° 53' 03.0000"	S 42° 52' 57.6165"
Longitude	E 147° 19' 19.0000"	E 147° 19' 23.9274"
Ellipsoidal height	100.000 m	77.163 m

Table 7-8: Sample input and output, using the Victoria/NSW Similarity parameters

	AGD66	GDA94
Latitude	S 33° 25' 25.12340	S 33° 25' 19.48962"
Longitude	E 149° 34' 34.34560"	E 149° 34' 38.58555"
Ellipsoidal height	603.345 m	610.873 m

Low Accuracy Transformation

Molodensky's Formulae

Excel Spreadsheet – Molodensky's Transformation

Molodensky's transformation method uses an average origin shift (at the centre of the earth) and the change in the parameters of the two ellipsoids. This method is often used in hand-held GNSS receivers with the old parameters published by United States National Imagery and Mapping Agency (NIMA) in their Technical Report 8350.2. These DMA parameters are now superseded by AGD66/84↔GDA94 parameters which have an estimated accuracy of about 5 metres:

Transformation from AGD66 or AGD84 to GDA94

Abridged Molodensky Formulae & Parameters

The United States Defense Mapping Agency previously published parameters for use with Molodensky's formulae, to convert between either AGD66 or AGD84 and WGS84 (DMA 1987). As for most practical purposes WGS84 is the same as GDA94, the same formula may continue to be used, but improved parameters are now available to convert between either AGD66 or AGD84 and GDA94 (AUSLIG 1997). It should be noted that these formulae require ellipsoidal height on input and give ellipsoidal height on output; however, the height component may be ignored if not required.

Note: *This transformation method should only be used for low accuracy projects (accuracy no better than 5 m). Other methods are available for higher accuracy projects.*

The AGD66/GDA94 parameters were derived from 161 points across Australia, which had both AGD66 and GDA94 coordinates, each of which also had a spirit levelled height. The AGD84/GDA94 parameters were similarly derived using 327 common points. These parameters are shown in Table 7-9.

Parameters

Table 7-9: Parameters – AGD66 & AGD84 to GDA94

	AGD66 to GDA94	AGD84 to GDA94
A	6378160 m	6378160 m
1/f	298.25	298.25
DX (m)	-127.8	-128.5
DY (m)	-52.3	-53.0
DZ (m)	152.9	153.4
Da (m)	-23	-23
Df	-0.00000008119	-0.00000008119

These parameters were tested using additional points with both AGD and GDA94 positions. A summary of these tests is shown in Table 7-10 and Table 7-11.

Table 7-10: AGD66 ↔ GDA94 parameters, residuals from 1262 points

	average (m)	std. dev. (m)	max. (m)	min. (m)
Latitude	-0.32	1.1	2.9	-5.9
Longitude	-0.56	0.9	3.3	-3.8
Ellipsoidal height	-0.97	1.4	3.8	-8.5

Table 7-11: AGD84 ↔ GDA94 parameters, residuals from 1588 points

	average (m)	std. dev. (m)	max. (m)	min. (m)
Latitude	0.70	0.7	5.2	-3.2
Longitude	0.41	0.4	5.3	-1.4
Ellipsoidal height	0.79	1.8	8.1	-4.4

Formulae

$$e^2 = 2f - f^2$$

$$\nu = a/(1 - e^2 \sin^2 \Phi)^{1/2}$$

$$\rho = a(1 - e^2)/(1 - e^2 \sin^2 \Phi)^{3/2}$$

$$\Delta\Phi(rad) = \{(-\Delta X \sin\Phi \cos\lambda - \Delta Y \sin\Phi \sin\lambda + \Delta Z \cos\Phi + (a\Delta f + f\Delta a)\sin 2\Phi)/\rho\}$$

$$\Delta\Phi" = 206264.8062\, \Delta\Phi$$

$$\Phi_{GDA94} = \Phi_{AGD} + \Delta\Phi$$

$$\Delta\lambda(rad) = \{(-\Delta X \sin\lambda + \Delta Y \cos\lambda)/(\nu \cos\Phi)\}$$

$$\Delta\lambda" = 206264.8062\, \Delta\lambda$$

$$\lambda_{GDA94} = \lambda_{AGD} + \Delta\lambda$$

$$\Delta h = \Delta X \cos\Phi \cos\lambda + \Delta Y \cos\Phi \sin\lambda + \Delta Z \sin\Phi + (a\Delta f + f\Delta a)\sin^2\Phi - \Delta a$$

$$h_{ANS} = H + N_{ANS}$$

$$h_{GDA94} = h_{ANS} + \Delta h$$

Examples

	AGD66	**GDA94**
Latitude	-37° 39’ 15.56"	-37° 39’ 10.18"
Longitude	143° 55’ 30.63	143° 55’ 35.43"
Ellipsoidal height	750 m	749 m
	AGD84	**GDA94**
Latitude	-37° 39’ 15.56"	-37° 39’ 10.17"
Longitude	143° 55’ 30.55"	143° 55’ 35.38"
Ellipsoidal height	750 m	748 m

Simple Block Shift

Excel Spreadsheet – Block Shifts Between AGD66, AGD84 & GDA94

A basic method of transforming by adding an average block shift in position determined from one or more common points. The accuracy of this transformation method is entirely dependent on the accuracy of the common point coordinates and the area over which the shift is to be averaged.

The average size of the block shifts between AGD66, AGD84 and GDA94 has been computed for 1:250,000 topographic map areas across Australia. These block shifts have a limited accuracy of about 10 m.

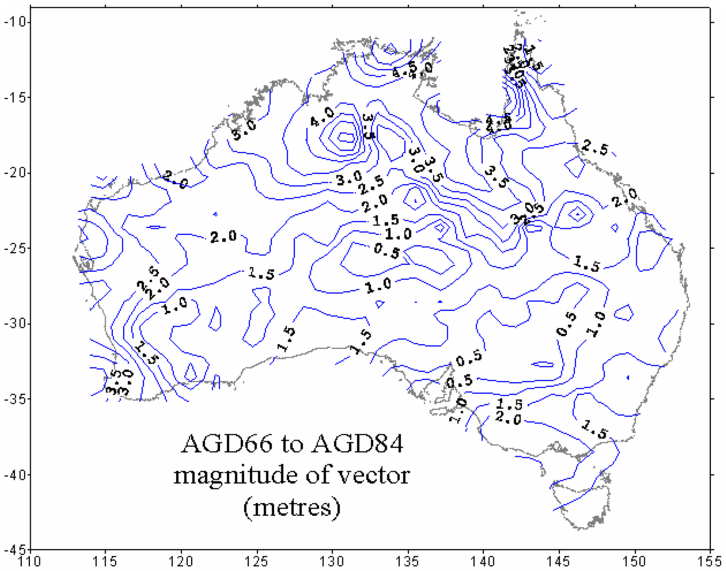


Figure 7-3: Magnitude of Block Shift vector, in metres, between ADG66 and ADG84

Comparison of Transformation Methods

The grid transformation is the recommended and most accurate method of transformation in Australia. Ideally no other method should be needed, but it is recognised that there are different user requirements, so less accurate transformation methods are also provided.

Comparison of transformation by various methods

The table below shows a sample of points that have been transformed from both AGD66 and AGD84 to GDA94 by the methods explained in this Chapter 7.

The 7 parameter (Similarity) transformation uses the appropriate national transformation parameters (AGD66 or AGD84). Similarly, the Molodensky transformation uses the AGD66 or AGD84 parameters as appropriate. The Grid Transformation uses the appropriate national grid (AGD66 or AGD84).

Table 7-12: Example transformation points from both AGD66 and AGD84 to GDA94 by various transformation methods

GDA Transformed									
	Known AGD84			Known GDA94	GDA94 (Block Shift)	GDA94 (Molodensky)	GDA94 (Similarity)	GDA94 (National NTv2 Grid)	Derived GDA94 Ellip. Ht. (AUSGeoid09)
Latitude	-29°	02'	52.0825"	47.6169"	47.64"	47.60"	47.602"	47.6175"	
Longitude	115°	20'	43.9092"	49.1004"	49.12"	49.04"	49.087"	49.1010"	
Ellip. Ht. (m)			284.998	241.291		240	242.46		241.329
N value (m)			18.5	-25.169					
AHD (m)			266.498						
Latitude	-20°	58’	57.9705"	53.1700"	53.15"	53.17"	53.181"	53.1667"	
Longitude	117°	05'	45.0683"	49.8726"	49.87"	49.86"	49.887"	49.8708"	
Ellip. Ht. (m)			129.902	109.246		108	109.05		109.191
N value (m)			14	-6.711					
AHD (m)			115.902						
Latitude	-19°	20'	56.0013"	50.4284"	50.44"	50.44″	50.432″	50.4303"	
Longitude	146°	46'	26.8165"	30.7906"	30.81"	30.75"	30.780"	30.7918"	
Ellip. Ht. (m)			541.709	587.077		588	583.64		587.152
N value (m)			13	58.443					
AHD (m)			528.709						
Latitude	-10°	35’	07.9983"	2.6077"	2.65"	2.67"	2.642"	2.6113"	
Longitude	142°	12'	35.5265"	39.5762"	39.54"	39.49"	39.524"	39.5716"	
Ellip. Ht. (m)			73.344	130.0452		136	129.99		130.176
N value (m)			14.6	71.432					
AHD (m)			58.744						
Latitude	-37°	23'	57.3212"	52.0181"	52.06"	52.05"	52.039"	52.0188"	
Longitude	140°	40'	45.4673"	50.4482"	50.47"	50.45"	50.440"	50.4497"	
Ellip. Ht. (m)			88.934	72.12		71	72.43		71.968
N value (m)			14.106	-2.860					
AHD (m)			74.828						
Latitude	-25°	42'	30.4392"	25.5493"	25.59"	25.58"	25.577"	25.5493"	
Longitude	122°	54'	29.7110"	34.6508"	34.64"	34.61"	34.640"	34.6509"	
Ellip. Ht. (m)			499.691	480.2142		479	479.68		480.489
N value (m)			10.79	-8.412					
AHD (m)			488.901						
Latitude	-17	31'	45.2316"	40.0554"	40.08"	40.08"	40.070"	40.0734"	
Longitude	128°	47'	56.4545"	0.98617"	00.99"	00.98"	00.988"	00.9781"	
Ellip. Ht. (m)			246.139	258.81		260	258.04		259.827
N value (m)			16.091	29.779					
AHD (m)			230.048						

Table 7-13: Example transformation points from both AGD66 and AGD84 to GDA94 by various transformation methods

GDA Transformed									
	Known AGD66			**Known GDA94**	**GDA94 (Block Shift)**	**GDA94 (Molodensky)**	**GDA94 (Similarity)**	**GDA94 (National NTv2 Grid)**	**GDA94 Ellip. Ht. (AUSGeoid09)**
Latitude	-42°	48'	22.3726"	16.9851"	16.93"	16.97"	16.963"	16.9846"	
Longitude	147°	26'	14.5257"	19.4355"	19.41"	19.49"	19.448"	19.4333"	
Ellip Ht (m)			64.76	41.126		42	44.45		41.139
N value (m)			20	-3.617					
AHD (m)			44.756						
Latitude	-18°	01'	37.8335"	32.7834"	32.77"	32.65"	32.659"	32.7833"	
Longitude	130°	39'	17.9193"	22.3169”	22.34"	22.37"	22.379"	22.3174"	
Ellip Ht (m)			348.195	363.34		365	362.43		364.088
N value (m)			17.225	33.118					
AHD (m)			330.970						
Latitude	-37°	57'	9.1288"	3.7203"	3.71"	3.73"	3.734"	3.7207"	
Longitude	144°	25'	24.7866"	29.5244"	29.467"	29.57"	29.555"	29.5258"	
Ellip Ht (m)			364.2	350.948		354	352.25		350.742
N value (m)			17	3.542					
AHD (m)			347.2						
Latitude	-37°	39'	15.5571"	10.1561"	10.81"	10.81"	10.176"	10.1563"	
Longitude	143°	55'	30.6330"	35.3839"	35.39"	35.43"	35.409"	35.3834"	
Ellip Ht (m)			761.986	749.855		749	750.05		749.627
N value (m)			17	4.641					
AHD (m)			744.986						

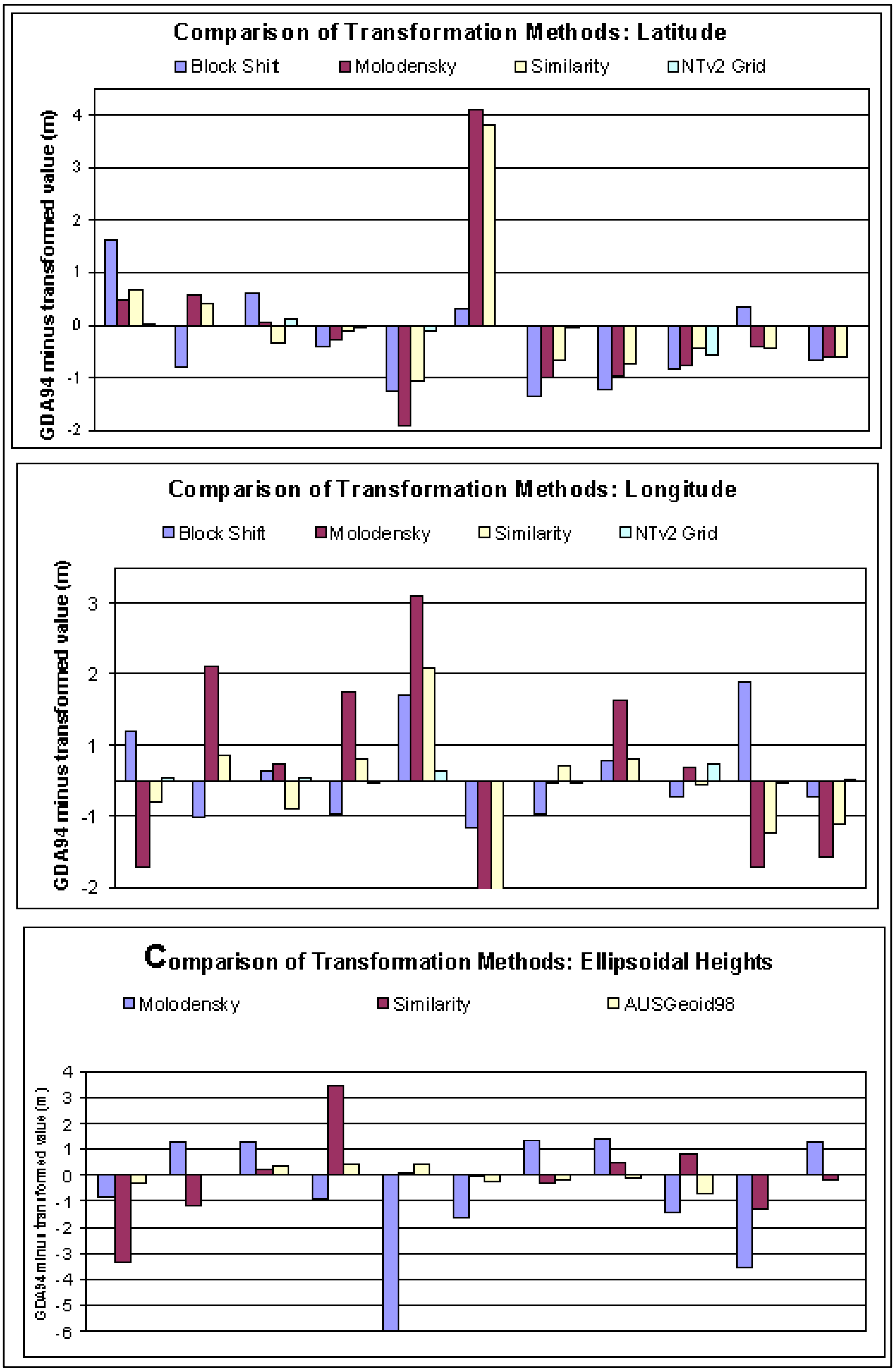


Figure 7-4: Comparison of transformation methods for latitude, longitude and height.

Chapter 8 The Australian Height Datum (AHD)

The Australian Height Datum (AHD) is the official height datum for Australia.

Background

On 5 May 1971 the then Division of National Mapping, on behalf of the National Mapping Council of Australia, carried out a simultaneous adjustment of 97,230 kilometres of two-way levelling. Mean sea level for 1966-1968 was assigned the value of zero on the Australian Height Datum at thirty tide gauges around the coast of the Australian continent.

The resulting datum surface, with minor modifications in two metropolitan areas, has been termed the Australian Height Datum (AHD) and was adopted by the National Mapping Council at its twenty-ninth meeting in May 1971 as the datum to which all vertical control for mapping is to be referred. The datum surface is that which passes through mean sea level (1966-1968) at the thirty tide gauges and through points at zero AHD height vertically below the junction points of the Basic Levelling (see below).

The determination of the AHD was documented in Division of National Mapping Technical Report No. 12 (Roelse, Granger et al. 1971).

Basic and Supplementary Levelling

Two-way levelling of third order accuracy or better, used in the original adjustment of 5 May 1971 which formed the AHD, is called "Basic levelling". Levelling subsequently adjusted to the AHD is called "Supplementary levelling".

Tasmania

The levelling network in Tasmania was adjusted on 17 October 1983 to re-establish heights on the Australian Height Datum (Tasmania). This network, which consists of seventy-two sections between fifty-seven junction points is based on mean sea level for 1972 at the tide gauges at Hobart and Burnie. Mean sea level at both Hobart and Burnie was assigned the value of zero on the AHD (Tasmania).

Islands

If the levels on islands closely adjacent to the Australian mainland are observed to standard third order accuracy, and are referred to mean sea level at a satisfactory tide gauge, they are deemed to be part of the Australian Height Datum.

AHD, Mean Sea Level and the Geoid

The AHD is an imperfect realisation of mean sea level because some of the tide gauges used for its definition were not in ideal locations; the mean sea level determination was for a limited period of time and no allowance was made for sea surface topography. The difference between AHD and mean sea level, which may be of the order of several decimetres (Mitchell 1990), is not significant for conventional propagation of AHD, which is relative to existing AHD bench marks, but may be important if connecting AHD to a recent determination of mean sea level.

Although the geoid is often equated to mean sea level, it may actually differ from it by the order of a metre, largely due to sea surface topography (Bomford 1980).

With improvements in geoid models and GNSS heighting, the difference between these three surfaces is sometimes apparent, particularly over large areas, or in areas where there are rapid changes in the slope of the geoid.

Chapter 9 The Australian National Geoid

The geoid is a surface of equal gravitational potential that is approximated by mean sea level. The height of the ellipsoid and the geoid can differ by tens of metres across the Australian mainland with the difference known as the geoid-ellipsoid separation (*N* value). Geoid – ellipsoid separation values can be used to reduce observed GNSS ellipsoidal heights (*h*) to heights relative to approximate Mean Sea Level (MSL).

AUSGeoid09

AUSGeoid09 is the latest in a series of national geoid models for Australia which provides the capability to convert from GNSS ellipsoidal heights to approximate AHD heights. These are known as derived AHD heights. AUSGeoid09 is a 1' by 1' grid (approximately 1.8 km) of AHD – ellipsoid separation (N_{AHD}) values. These N_{AHD} values have an estimated uncertainty of ±0.06 m at the 95% confidence interval (Brown et al. 2011). In most cases, AUSGeoid09 provides a relative uncertainty comparable to $[12\ \mathrm{mm} * \sqrt{k}]$ where k is distance in km for relative height transfer.

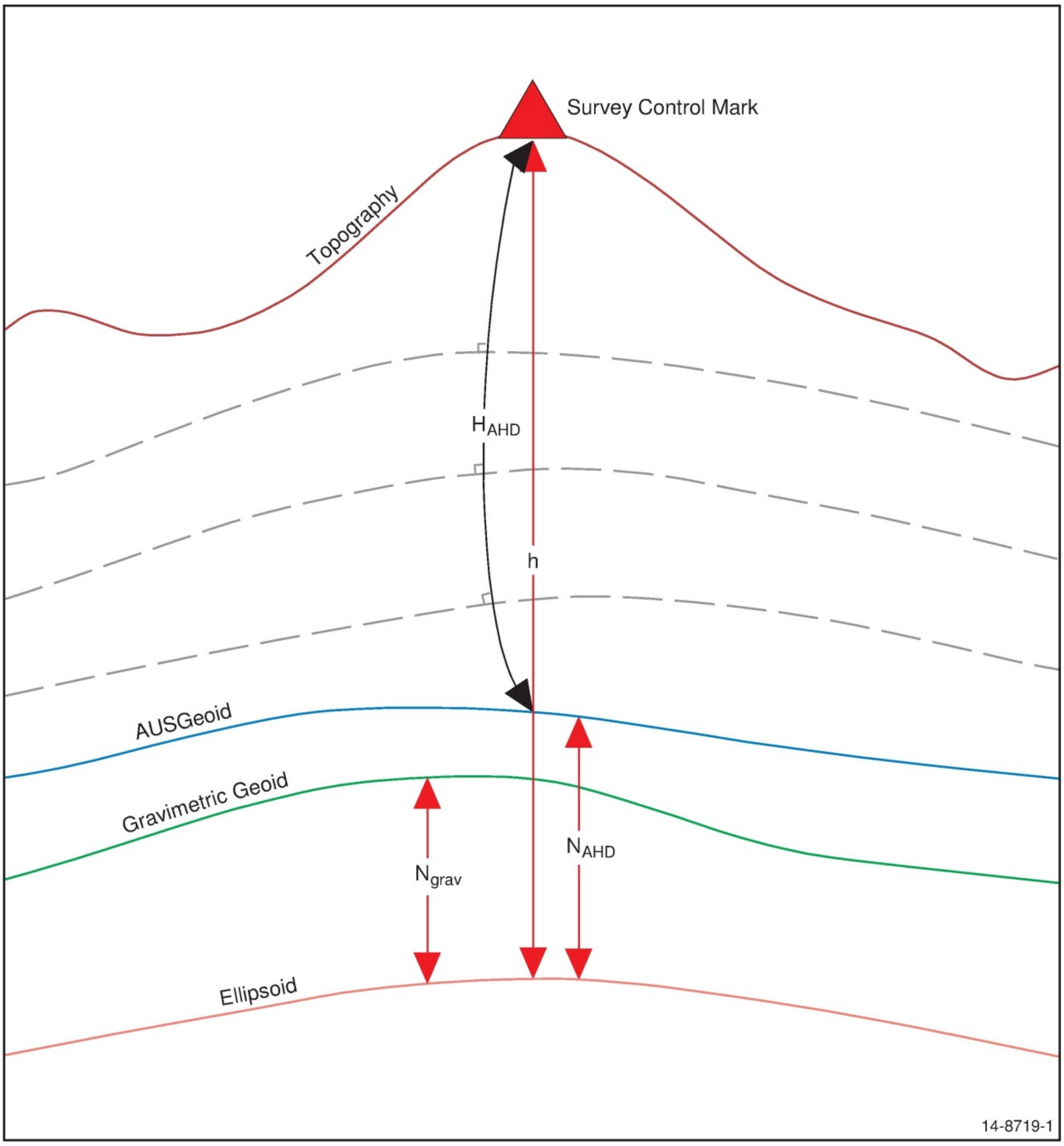


Figure 9-1: Relationship between the ellipsoid, gravimetric geoid, AUSGeoid, and the topography.

$$H_{AHD} = h - N_{AHD}$$

where:

$$H_{AHD} = \text{derived AHD height}$$
$$h = \text{ellipsoid height}$$
$$N_{AHD} = \text{AUSGeoid09 value}$$

In places where collocated AHD and GNSS data were not used in the development of AUSGeoid09, discrepancies between the derived AHD value and the published AHD value (from a jurisdictional database) can occur. The cause of these discrepancies includes uncertainty in the observed ellipsoidal height and local / regional deformation since the time of the AHD levelling.

Prior to AUSGeoid09, the AUSGeoid models were largely gravimetric geoid models, and sea surface topography effects caused errors of up to ±0.5 m between derived and published AHD heights. Given that this error was predominantly caused by a 1 m trend from south-west to north-east Australia, these discrepancies between the derived AHD value and published AHD values were minimised by applying the N values differentially, rather than in an absolute sense. AUSGeoid09 provides a more direct and more accurate model for converting ellipsoidal heights to derived AHD heights by incorporating a geometric component which models the offset between the gravimetric geoid and AHD.

AUSGeoid09 data files and interpolation software and further information can be obtained from Geoscience Australia's web site.

AUSGeoid09 Technical Specifications

AUSGeoid09 is a gravimetric – geometric quasigeoid. The gravimetric component (Featherstone et al. 2011) was developed using:

- degree-2160 spherical harmonic expansion of the EGM2008 global gravity model (Pavlis, Holmes et al. 2008)
- approximately 1.4 million land gravity anomalies from the Australian national gravity database
- the 9” x 9” GEODATA-DEM9S digital elevation model of Australia, and
- altimeter-derived marine gravity anomalies from the DNSC2008GRA grid (Andersen, Knudsen et al. 2010)

The geometric component (Brown et al. 2011) was developed using two datasets:

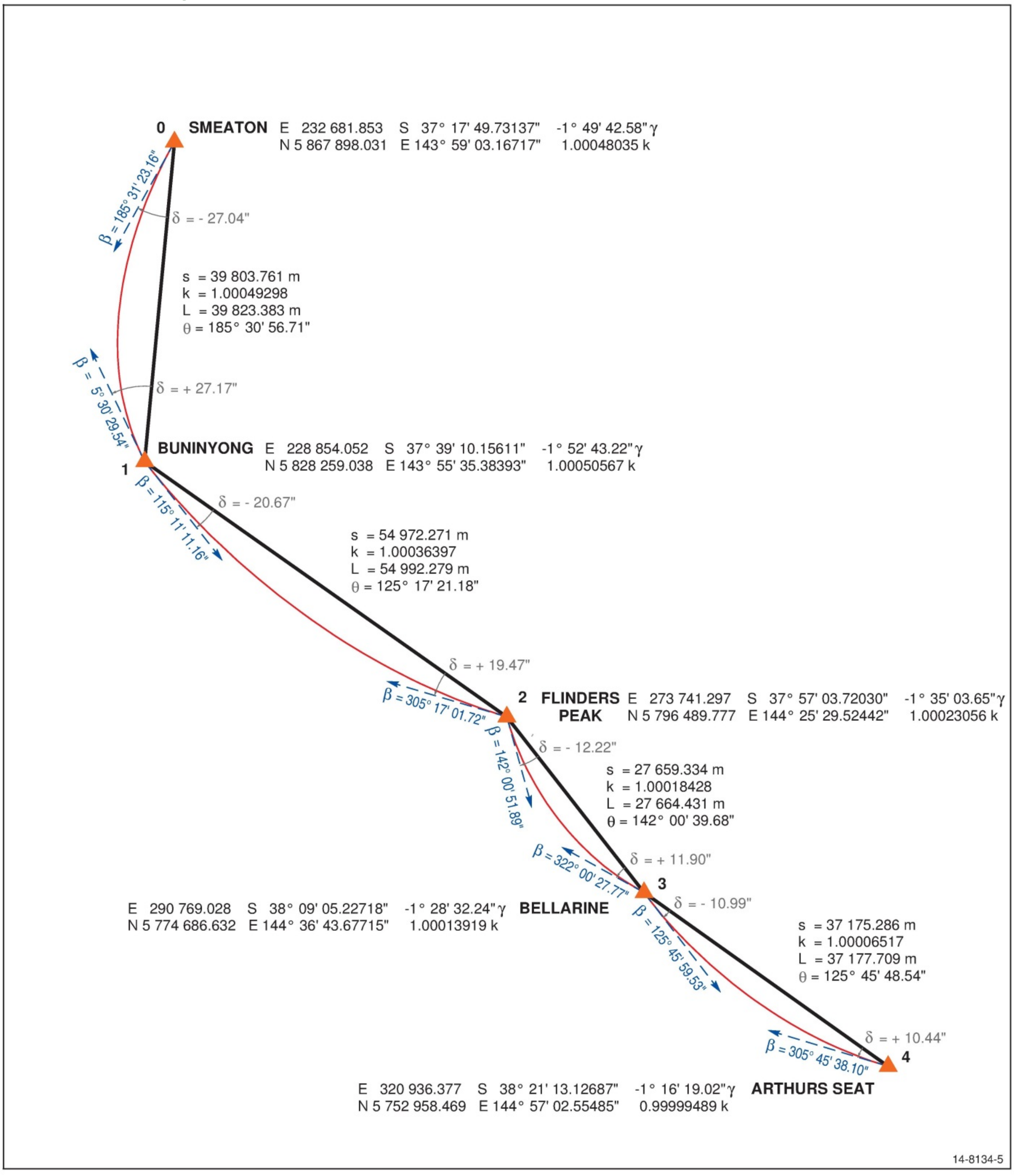
- The primary dataset of 2638 co-located GNSS-AHD heights provided by the State and Territory surveying authorities.
- A secondary dataset of 4233 Australian National Levelling Network Junction Points to provide a higher-resolution definition of the offset between the AHD and gravimetric quasigeoid.

Chapter 10 Test Data

GDA94 and MGA94 (zone 55) values

	Buninyong	Flinders Peak
Latitude (Φ)	S 37° 39' 10.15611"	S 37° 57' 03.72030"
Longitude (λ)	E 143° 55' 35.38393"	E 144° 25' 29.52442"
AHD (H)	744.986 m	347.200 m
$N_{AUSGeoid09}$	4.641	3.542
Ellipsoidal height (h)	749.855 m	350.948 m
Easting	228,854.052	273,741.297
Northing	5,828,259.038	5,796,489.777
X	-4,087,103.458	-4,096,088.424
Y	2,977,473.0435	2,929,823.0843
Z	-3,875,464.7525	-3,901,375.4540
Azimuth (α)	127° 10' 25.07”	306° 52' 05.37"
Grid convergence (γ)	-01° 52' 43.22"	-01° 35' 03.65"
Grid bearing (β)	125° 17' 41.86"	305° 17' 01.72"
Arc to chord (δ)	-20.67"	+19.47"
Plane bearing (θ)	125° 17' 21.18"	305° 17' 21.18"
Point scale factor (k)	1.000 50567	1.000 23056
Meridian distance (m)	-4,173,410.326	-4,205,192.300
Rho (ρ)	6359253.8294	6359576.5731
Nu (ν)	6386118.6742	6386226.7080
Ellipsoidal distance (s)	54,972.271	
Line scale factor (K)	1.000 36397	
Grid (& plane) distance (L)	54,992.279	
Meridian convergence (Δα)	18' 19.70"	
Line curvature (Δβ)	40.14"	

Traverse Diagram



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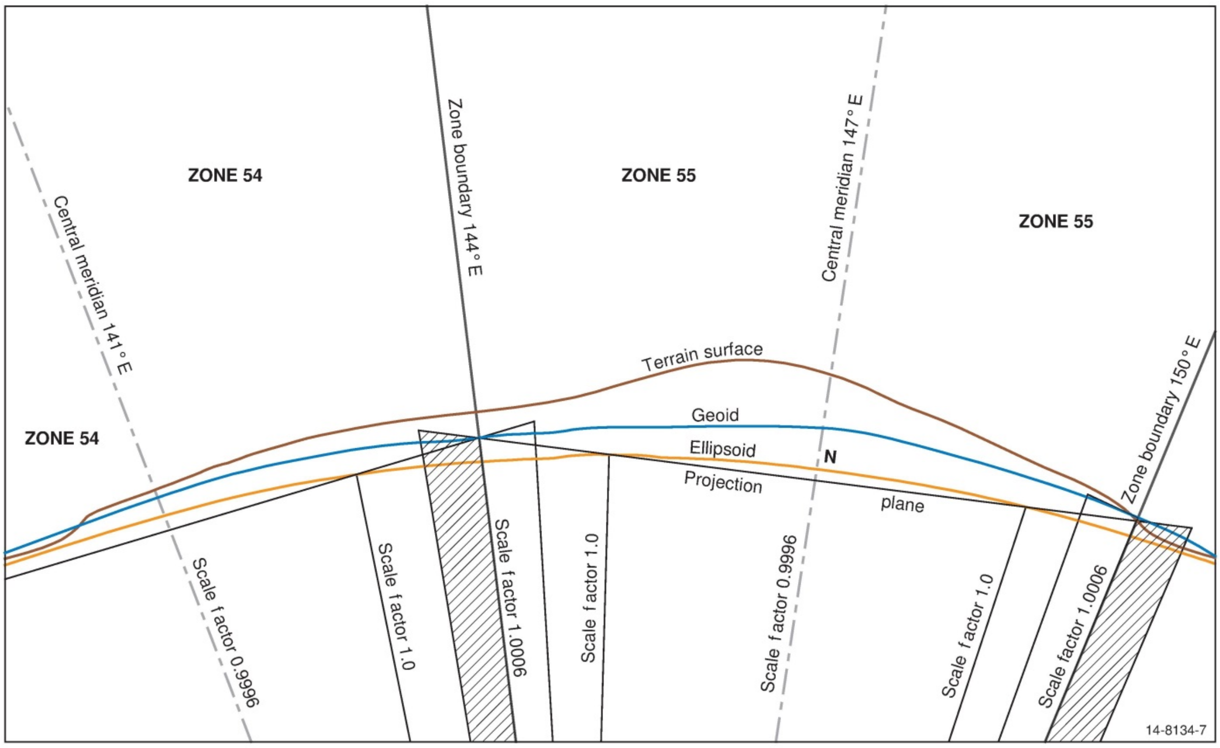
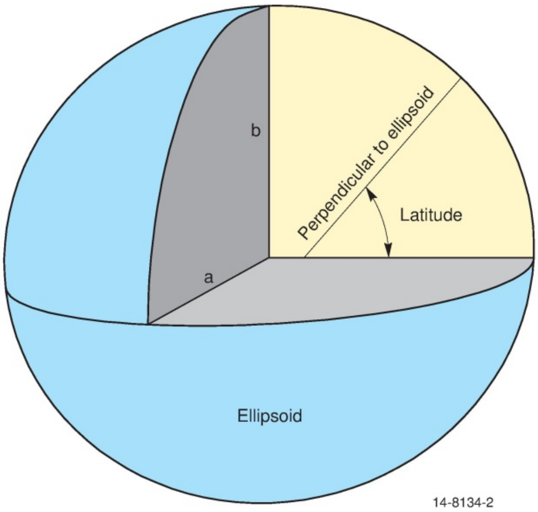
Diagrams



MGA cross section



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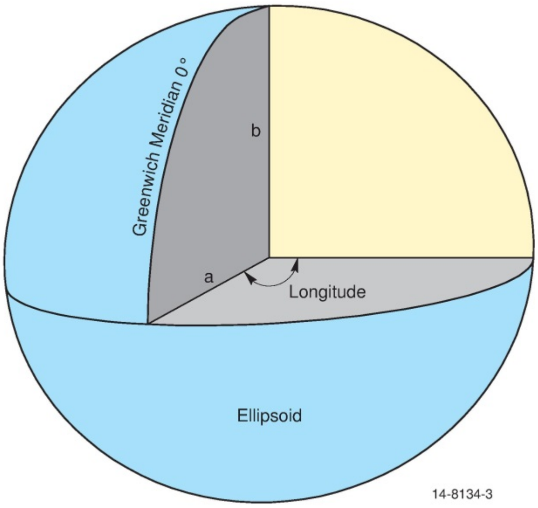
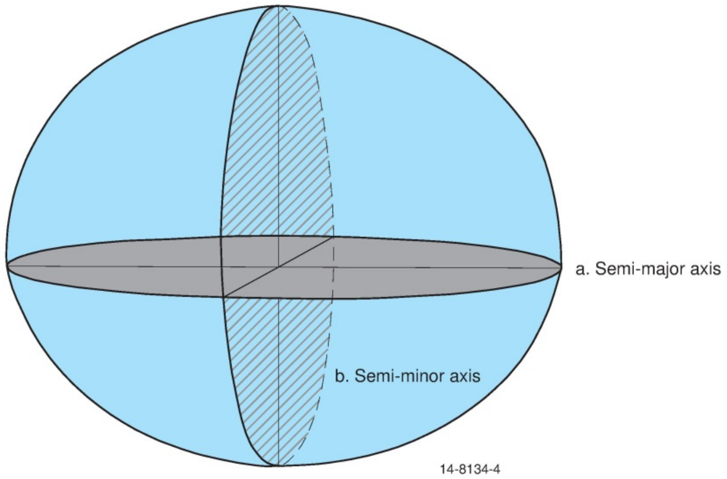
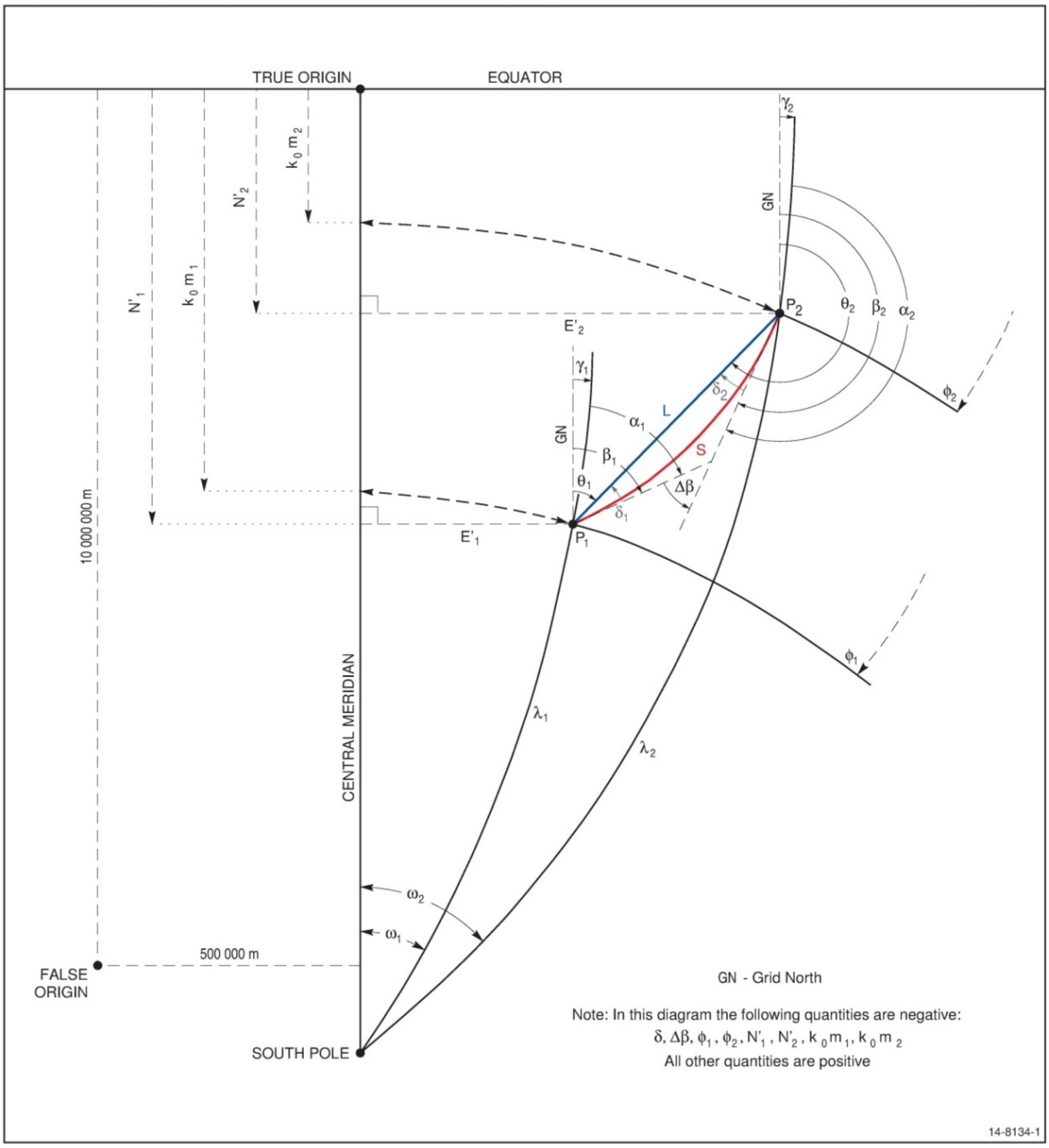


Longitude



Semi-major and Semi-Minor axis



UTM Projection

Greek Alphabet

Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	Ξ	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	φ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω