William Dawes' Gravity Measurement in Sydney Cove, 1788

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SUMMARY

William Dawes arrived in Australia in January 1788 as an astronomer with the Australian First Fleet and as the Board of Longitude's official observer. During his time in Australia he carried out many astronomical observations, of which the record has gone lost. The fieldbooks were possibly still with the widow of William Wales, of the Board of Longitude, after Wales died.

What has not been lost are his gravity observations in Sydney Cove in Australia, of 1788, made with a temperature compensated grid iron pendulum, of which a record can be found in his correspondence with Nevil Maskelyne, the English Astronomer Royal.

As far as I know, William Dawes' pendulum gravity observations have not been published previously as such, until the recent paper by Morrison and Barko (2009). I helped investigate this series of observations which led to the first gravity acceleration determination on Australian soil, of which the record has survived. This paper reports on my analysis of his precision pendulum gravity determination. In this story, William Wales speaks from his grave, in support of Dawes.

RESUMEN

Guillermo Dawes llegaba a Australia en el Enero de 1788, como astrónomo con la Flota Primera Australiana y como el observador oficial del Consejo de la Longitud de la Inglaterra. Durante su tiempo en Australia el fue cargado con hacer muchos observaciones astronómicas, pero los libros de anotaciones fueran perdidos porque ellos todavía estaban posiblemente con la viuda de Guillermo Wales, del Consejo de la Longitud, después del muerte de Wales.

Pero lo que no fue perdido, son sus anotaciones de determinaciones gravimétricas de 1788, con péndulo compensado de rejilla en el Sídney Cove, Australia, porque todavía los podemos encontrar en su correspondencia con Nevil Maskelyne, el Astrónomo Real, de la Inglaterra.

No creo que las observaciones gravimétricas péndulares de Guillermo Dawes fueran interpretado correctamente como algo gravimétrica, hasta que un publicación recienta por Morrison y Barko (2009). Yo ayudé con la investigación de este serie de observaciones, que resultó en la primera determinación de la aceleración de gravitación sobre la tierra de Australia, de que todavía tenemos las observaciones. Esta reporta aquí, presenta mi estudio de sus determinaciones gravimétricas péndulares de precisión. En esta historia, Guillermo Wales habla desde su tumba, soportando al Guillermo Dawes.

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Recently a paper by Morrison and Barko appeared in the Journal *Historical Records of Australian Science*. This paper was named: *Dagelet and Dawes: Their Meeting, Their Instruments and the First Scientific Experiments on Australian Soil*. In the course of the development of their paper, in which the January 1788 meeting was discussed between Joseph Lepaute Dagelet (d'Agelet was the La Pérouse expedition's astronomer) and William Dawes (the astronomer with the Australian First Fleet), I helped investigate what could well be the first gravity determination on Australian soil of which the record has survived, observed by William Dawes. It was suggested to me to write a record of my investigation of Dawes' pendulum gravity observation, and demonstrate his gravity value.

1. Historical description

In 1788 a gravity observation generally consisted of a series of daily clock rate comparisons of a special pendulum clock, timed against some astronomical observations (daily equal altitudes of the sun either side of midday) often aggregated over the course of a whole month (Table 1). The pendulum had to have a precisely determined length that could be easily reproduced anywhere at the 10 micron level and the pendulum had to be of a temperature compensated type. When observing gravity via a pendulum, an amplitude dependent correction has to be made to the clock rate necessitating also the recording of the pendulum half amplitude, which the observers referred to as "the arc from the vertical" or "the arc of vibration".

The correction is usually calculated by using a series expansion of what we now call an elliptical integral and this correction was known to Nevil Maskelyne, the English Astronomer Royal. He had all his observers record this "arc from the vertical" during their voyages, together with the clock rate, for an agreed length of the pendulum. The full pendulum equation can be found on page 335 in Giancoli (1988):

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{2^2}\sin^2\frac{\theta_M}{2} + \frac{1}{2^2}\frac{3^2}{4^2}\sin^4\frac{\theta_M}{2} + \cdots\right)$$

where θ_m is the "arc from the vertical", L is the length of the pendulum in metres, and g is the acceleration of gravity in m/s^2 and T is the period of a full swing of the pendulum in seconds.

So $T^2 = 4\pi^2(L/g)(1+\Delta)^2$ where $(1+\Delta)^2$ is the above series between the brackets, squared.

First a description of the instrument: the astronomical regulator clock. The astronomical regulator clocks carried around the world by the astronomers had a "seconds" pendulum, which was almost a metre long and which "escaped dead seconds in the manner of the late Mr Graham" (Wales & Bayly, 1777). This pendulum was temperature compensated after an idea of George Graham (1673 – 1751) to use two different types of metal. The pendulum was symmetrically folded (fig. 4), so that the expansion of one type of metal lifted the pendulum bob and the expansion of the other type of metal lowered the pendulum bob. The ratio of the expansion coefficients of the two types of metals had to be in a certain balance with the lengths of the different metals so that the whole thing becomes temperature compensated. It was the handiwork of John Harrison (of *Longitude* fame) that put this idea into practice (fig. 5). As the resulting pendulum length needed to be adjustable with very high precision, it was provided with a regulator nut. The regulator clock acquired its name from this fine motion "screw". At the bottom of the pendulum you find a nut with a graduation engraved around it, and it was generally constructed in such a way that a full turn of the nut altered the rate of such a clock by very close to half a minute per day.

2. Defining the problem

The pendulum length used by William Dawes now needs to be determined.

When describing his pendulum, Dawes writes on Oct 1st, 1778: "the screw is at 15 on the nut" (Morrison, 2008). In order to understand the meaning of this, we find a statement on page 131 by William Wales in Wales & Bayly (1777) in his record of astronomical observations of Cook's second voyage, where he describes the setting of the pendulum length:

Quote: .....it was always altered, in order to its being packed up, yet on setting up again, it was constantly brought back to its proper length, by means of a scratch on the rod, and the numbers on the nut.
Unquote.

In his "Introduction" to the observational record of the voyage, William Wales writes in Wales & Bayly (1777):

Quote: ......On reconsidering the circumstances of the clock's different rates of going at the Cape of Good Hope in November 1772 and April 1775, I am rather inclined to alter my opinion (see page 131) and to conclude that I made a mistake in setting the pendulum to its proper length, either when here in November 1772 or at Dusky Bay in New Zealand, after which time it was never altered; basically as the difference corresponds nearly to that which would arise from a whole revolution of the nut which supports the ball of the pendulum, namely 28" or 29", increased by the same quantity that the clock had gone faster on being set up a second time both at Point Venus and Queen Charlotte's Sound..... Unquote.

In the left figure, fig. 1 below, from Howse & Hutchinson (1969), we can see the top part of a round pendulum bob, and an example of such a set of scratch marks on the rod, to which the bob can be set. The last adjustment is made by setting the regulator nut to a predetermined number on the nut. This nut is visible just below the bob on the picture to the right (fig. 2). You can see a little flat bit directly above the regulator nut: possibly an index to aid the setting of the nut. Underneath the pendulum is a larger graduated plate with which one can make repeat readings of the pendulum amplitude value, the "arc from the vertical", in order to get a good mean value.

The clock that William Dawes had was a Shelton astronomical regulator clock (similar to fig. 5), which was of a type also carried on the three James Cook voyages. If we would

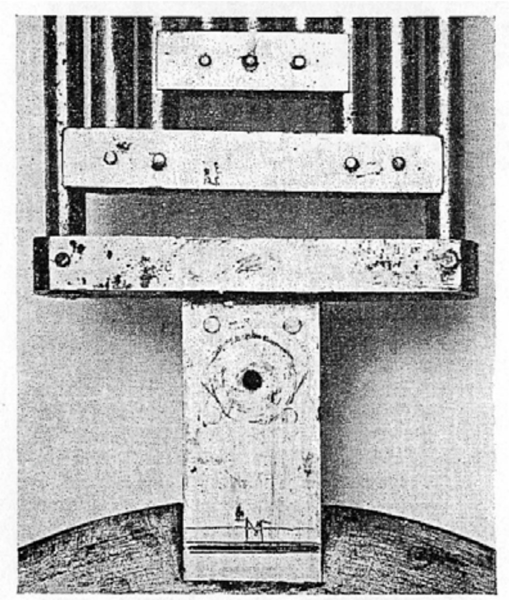



Fig.1 After Howse&Hutchinson (1969), left. Fig.2 After Nick Lomb (2004), right.

possess data on the length of Dawes' pendulum, like Nevil Maskelyne did, we would be able to calculate his 1788 gravity value for Sydney Cove. In this paper I show how to do just that, with help from William Wales.

3. Observations

William Dawes has left us with correspondence to Nevil Maskelyne which gives us the daily rate of going of his Shelton clock at Sydney Cove for a section of September 1788. Laurie (1988) shows that in a letter of the 1st of October 1788, Dawes mentions the regulator clock losing 36 seconds in sidereal time in one sidereal day, and in the last 11 days before October 1st the clock was losing 37.25 seconds per day (Morrison, 2008). There were many more comparisons carried out by him in the following months and years (there is a list of dates of these, in other correspondence) for the going of the clock, but the relevant field books have been lost when in possession of William Wales, who died in 1798. Laurie (1988) also shows that on 16th of April 1790 Dawes wrote to Nevil Maskelyne that the pendulum's "arc of vibration" has been 1 degree and 30 sec from the beginning and continued to do so "constantly the same". In order to calculate a value for the acceleration of gravity, we need to know one more variable: the pendulum length. Thanks to the work by William Wales this is possible.

Obſervations at St. Peter and Paul Oſtrog at Kamtſchatka.

Equal Altitudes for the Going of the Clock N° 2.

1779.	Time of Noon per Clock uncorrect. H. ′ ″	Half Interval. H. ′ ″	Time of Noon per Clock correct. H. ′ ″	Clock ſlow for Sidereal Time. ′ ″	Daily Rate of Clock getting. ″	N° of Obſervations	Phenomena and Remarks.
♃ May 13.	3 16 59,8	4 40 36	3 16 44,8	1 48,9		25	Sun.
♀ —— 14.	3 21 6,8	4 48 54	3 20 51,7	1 37,5	11,4	22	Do.
☉ —— 16.	3 29 25,2	3 52 48	3 29 13,0	1 9,0	14,2	14	Do.
♃ —— 20.	3 46 14,4	5 20 30	3 45 59,9	0 15,6	13,35	22	Do.
♀ —— 21.	3 50 23,0	4 7 6	3 50 12,4	0 2,12	13,48	23	Do.
☉ —— 23.	3 58 51,6	4 12 36	3 58 40,5	Faſt. 0 25,5	13,81	8	Do.
☿ —— 28.	4 20 6,0	3 2 30	4 19 58,5	1 31,4	13,20	18	Do.
☽ —— 31.	4 33 1,4	5 20 3	4 32 51,4	2 13,2	13,93	26	Do.
June 8.	By computing the time from ſingle altitudes taken this morning, the Clock was 3′ 55″,2 too faſt for ſidereal time at noon, and getting at the rate of 12″,75 per day on ſidereal time ſince the 31ſt of May. The pendulum vibrated from 1° 24′ to 1° 25″, which is 12 ſeconds leſs than the arc of vibration on each ſide (o) when a going in a low latitude where the weather is much warmer.						

Equal Altitudes for the Going of the Clock N° 2, when at St. Peter and Paul the ſecond time.

1779.	Time of Noon per Clock uncorrect. H. ′ ″	Half Interval of Obſervations. H. ′ ″	Time of Noon per Clock correct. H. ′ ″	Clock faſt for Sidereal Time. ′ ″	Daily Rate of the Clock. ″	N° of Obſervations	Phenomena and Remarks.
♃ Aug. 26.	10 18 38, 1	4 58 1	10 19 1,56	0 48,06	Getting.	14	Sun.
♀ Sept. 1.	10 41 49, 1	4 16 54	10 42 10,70	2 1,2	12,18	25	Do.
♃ —— 2.	10 45 40, 0	4 5 2	10 46 0,29	2 13,79	12,59	20	Do.
♄ —— 4.	10 53 18, 0	5 0 0	10 53 42,68	2 41,83	14,02	16	Do.
☉ —— 5.	10 57 7, 6	4 54 42	10 57 31,90	2 54,40	12,57	16	Do.
☽ —— 6.	11 0 57,75	4 46 0	11 1 21,75	3 6,85	12,45	12	Do.
♂ —— 7.	11 4 48,18	4 51 12	11 5 12,84	3 20,54	13,69	17	Do.
☿ —— 8.	11 8 38,41	4 43 30	11 9 2,77	3 34,47	13,93	14	Do.
♃ —— 16.	11 39 27,59	3 45 48	11 39 50,35	5 36,05	15,07	12	Do.
☉ —— 19.	11 51 1, 0	3 9 20	11 51 22,62	6 22,37	15,44	11	Do.
♀ —— 24.	12 10 14,09	4 26 6	12 10 36,39	7 37,09	14,94	12	Do.
♄ —— 25.	12 14 4,97	3 30 48	12 14 27,83	7 52,00	14,91	20	Do.
☉ —— 26.	12 17 53,48	4 12 6	12 18 17,93	8 7,10	15,10	16	Do.
27.	12 21 42,74	4 2 7	12 22 9,69	8 21,86	14,76	12	Do.
	The pendulum vibrated from 1° 31′¼ to 1° 32′ on each ſide (o), which is 7″ more than before; this ſeems owing to the weather being much warmer.						

Table 1. A page from Bayly (1782), showing a typical gravity observation from "the Going of the Clock".

4. Error budget

The variables in the pendulum equation are the swing period, the pendulum length and the arc of vibration. The swing period was determined by comparison with equal altitude observations of the sun, either side of midday for determining local noon. The quality of these clock rate observations were important so a large number of repeats were observed averaged to 1/100-th of a sec. We can conservatively credit them with an 0.1 second per day time base or about 1 ppm. But other factors influence the result. Resonance of the axis of motion of the pendulum was countered by chocking the clock with guy posts. The pendulum amplitude also needed observing: it is influenced by the rust level or stiffness of the pendulum suspension spring, the level of winding of the clock, ambient temperature, etc. The pendulum motion is not precisely a simple harmonic motion. At increasing pendulum amplitude the clock rate is not totally independent anymore of the amplitude and the relationship is not linear but quadratic. This correction already reaches 9.5 parts per million when the arc from the vertical is half a degree. As this angle is usually the triple or almost quadruple of this, the correction increases to between nine-fold to sixteen-fold. At an average swing of the pendulum, an accuracy of 5 minutes of arc in this amplitude estimate affects the calculated gravity reading by say 10 parts per million.

The pendulum length has been temperature compensated as mentioned above (say to 5 ppm), by the use of metals with different expansion coefficients. With the setting accuracy of the pendulum length at about one third (or better) of one of the 28 engraved divisions on the regulator nut, we get a potential length resolution of 0.0003 inch (7.5 microns). This is phenomenal and is about 10 parts per million or less, of the pendulum length. Figure 3 shows an example of an engraved modern regulator nut.

Fig.3 Courtesy Artur Kucharczyk

5. Methodology for retrieval of the pendulum length

It would be expected that William Dawes used the London pendulum length. This can be verified by consulting relative gravity between Sydney and London, by using a normal gravity model and a chart of anomalies. I investigated whether the London pendulum length had been used by him and concluded to my dismay that this was not so: his daily clock rate seems to be 30 sec too fast for that, it should have lost over a minute per day.

I thought I'd investigate other contemporary gravity observational work. Howse (1969) shows a table of clock rates for Cook's second voyage, but without the crucial "arc from the vertical" data. Luckily I found that we have enough other information to draw a conclusion on the pendulum length of Dawes' regulator clock, by studying William Wales' gravity work in Wales & Bayly (1777). I went to the State Library of NSW and investigated the raw observations of Captain James Cook's second voyage, as published in 1777 by his astronomer William Wales, in order to solve the puzzle by accessing William Wales' "arc from the vertical" observations, not listed in Howse (1969).

The late William Wales has now indeed come to the rescue, when he explains some discrepancies in his clock rates. Thanks to William Wales' own review of problems experienced by him in setting the pendulum length during Cook's second voyage, I have come up with an answer for the other pendulum length, the one from William Dawes.

At first sight the following exposé looks circular, but it is based on solving for a contemporary pendulum length estimate, using a larger population of another's appropriate pendulum observations (those of William Wales) and checking whether there is any supporting evidence for the end result.

6. Data Cleaning

The discrepancies in William Wales' clock rates were mentioned above. To complicate matters, the London pendulum length appears not to have been used by William Dawes. This now needs investigation.

During Captain Cook's third voyage, Lieutenant King writes among his observations that he found the nut on the pendulum had shifted and he reset it from 28 (0) to 2, which made the going of the clock two seconds per day faster for an increase in the nut setting against the index (Bayly, 1782). To run faster the pendulum has to be shorter, from this I can conclude that the nut was engraved with a clockwise graduation if the thread of the pendulum is of a normal handedness type. It can be set with an accuracy of half to a third of a division engraved on the nut, but the standard practice was to set it to a full integer.

From the remark by William Wales in Wales & Bayly (1777), that a full turn of the nut changed the rate of going of the clock by 28 to 29 seconds per day (let's say 28.5") and from a remark from Lieutenant King saying that a setting of 28 was equal to "(0)" in Bayly (1782), I can calculate an estimate of the speed of the pendulum regulator thread:
Using the simple pendulum equation, we can see that the ratios of the pendulum lengths of William Wales before and after the mistake (a whole turn of the nut) are proportional to the ratios of the squares of the periods, if all else is kept the same:
$L_0/L_1 = T_0^2/T_1^2$ or $L_0 = L_1 * T_0^2/T_1^2$ where L_1 and T_1 are the length and the period with the mistake.
So the length change is $L_1 - L_0 = L_1 - (L_1 * T_0^2/T_1^2) = L_1 (1 - T_0^2/T_1^2)$
The ratio of the periods (without and with 28.5" mistake) is 86400/86428.5 or 0.9996702 as there are 86400 seconds in a day, and squared this ratio gives a value of 0.999341.
The change of the pendulum length $L_1 - L_0$ is derived from 1 minus this squared ratio, scaled by the general length of the seconds pendulum, described by George Graham (Nicholson, 1825) as about 39.13 inches or g/π^2. This gives me a change to the pendulum length of 0.0258 inches for a full turn of the nut. Now we know how to clean Wales' data! Also, the speed of the thread is the inverse which estimates at 38.8 threads per inch.

7. Results

Where is this all leading to? I have used a normal gravity model and relevant anomaly map for an estimate of the gravity at all the harbours in Cook's second voyage where William Wales observed the rate of going of his pendulum clock. I derived the normal gravity formula for EGM2008 and checked with the equivalent 1967 International Formula for normal gravity (Heiskanen & Moritz, 1967). I used a EGM2008 anomaly chart on the web with the first, and a crude 1967 anomaly chart (Vanicek, 1986) with the latter. Using the clock rates and pendulum amplitudes I solved for the pendulum lengths for these harbours and found that at the harbours affected by the above mistake which Wales mentioned, he had an average effective pendulum length of 39.111 inches, with 0.002 inch as the standard deviation of the mean (Table 2).

When we take the above quote from William Wales in account, in which he says his clocks were running too fast commensurate with a full turn of the nut, we can conclude that the effective pendulum length setting was 0.0258 inch shorter than intended, which would speed up the clock by the required number of seconds mentioned by Wales. Together with the solved pendulum length of 39.111 inches, this then gives us an estimate of about 39.137 inches as the setting of the pendulum (the London pendulum length) which was intended by Nevil Maskelyne and which would give the clock a zero daily rate of going in London when rated against the astronomically observed results there.

Now that we can conclude that the intention was to maintain the London pendulum setting as the length, we would expect William Dawes' pendulum to be set at this length as well. But using the clock rate reported by William Dawes (37.25 sec slow in Sydney Cove) and the ratio of normal gravity for Sydney and London, corrected with an appropriate anomalies chart, we can conclude that William Dawes' effective pendulum length would not give a zero daily clock rate gain or loss in London, but actually would be thirty seconds fast per day in London and thus have the wrong pendulum length selected (a whole turn of the regulator nut):

The ratio of the square of the pendulum periods in two places is inverse to the ratio of gravity in the two locations if all else remains the same; this can be worked out from the pendulum equation.

$$T_L^2/T_S^2 = g_S/g_L \text{ which can be expressed as } T_L = T_S\sqrt{(g_S/g_L)}$$

The general period for a seconds pendulum is two seconds. If the pendulum runs slow in Sydney by 37.25 sec in a day, its period will be larger than two seconds by the ratio 86400/(86400 - 37.25), which gives us a Sydney pendulum period T_S of 2.00086264 sec for a forward swing plus a backward swing. For the Greenwich latitude of 51.17 decimal degrees and 33.87 for Sydney we get a normal gravity ratio γ_S/γ_L of 0.998438, using the equation for normal gravity. The square root of this is 0.999219. This means the London pendulum period $T_L = T_S\sqrt{(\gamma_S/\gamma_L)}$ would be 1.999300 sec or 0.999650 times 2 sec. In 86400 true seconds the same pendulum would beat 86400 / 0.999650 or 86430.25 sec in London, which means the clock would run 30 seconds fast per day in London. This proves William Dawes did not have the London pendulum length set on his astronomical regulator clock in Sydney.

This suggests to me that he made the same mistake as William Wales (and William Bayly) did during Cook's second voyage and set the pendulum nut wrong by a whole turn of the nut. Dawes effectively shortened the intended pendulum length of 39.137 inches by 0.0258 inch to the same 39.111 inches as occurred with William Wales; this would indeed make the clock gain 30 seconds per day in London.

So, armed with this resolved pendulum length I can now combine William Dawes' astronomical regulator clock rate and his 1.5 degree arc from the vertical reading as follows and find a gravity value for Sydney Cove, observed by him in 1788, using the above pendulum period equation from Giancoli (1988).

We can rework this pendulum equation so that it expresses gravity as a function of the other variables:

$$g = 4\pi^2.L(1 + \Delta)^2 / T^2$$

where the squared series expansion $(1 + \Delta)^2$ works out as 1.000086 for $\theta_m = 1.5$ degrees. Above we found that T = 2.00086264 for Dawes' pendulum. After converting the pendulum length L of 39.111 inches into metres (0.9934194 m) we see that the Sydney Cove gravity measurement results in g = 9.79705 m/s^2, or **979.705 gal**. This has an accuracy of about 25 parts per million, observed in September 1788 by William Dawes, with local noon determined by equal altitude observations of the sun, through most of September, in order to determine his clock rate.

8. Analysis

I first thought that the only explanation for the short 39.111 inch pendulum lengths was that Nevil Maskelyne must have made early use of a pendulum length for a standard latitude of 45 degrees. Around 1790 there was a move before the French National Assembly and also before the American Congress to consider a standard latitude of 45 degrees for the choice of pendulum length for science. I reported this to Barko and Morrison when they were preparing their above mentioned paper on the meeting between Dawes and d'Agelet, but a later inspection recently by me of the raw observations as described by William Wales in Wales & Bayly (1777) and his explanation of his mistake, set me straight.

We can assume this 39.137 value, which I estimated via the thread speed, to have been the Greenwich calibration value for an astronomical regulator clock pendulum length during Cook's second voyage. Indeed, later in 1819, according to Zupko (1990), there is evidence of 39.1372 inches being used as the standard pendulum length setting.

According to Derek Howse (Howse, 1969), we can find in Board of Longitude papers that William Wales had the index to the bob of his pendulum stand at 13 on the nut and he had the top of the bob come up against a horizontal scratch on the rod, when going at the Royal Observatory at Greenwich (March 28 to April 1st, 1772), and the clock gained 5.03 seconds per day against astronomical obs. Sixteen years later William Dawes confirms that the pendulum of his particular Shelton astronomical regulator clock stood at 15 on the nut. What seems to have happened to them both is that when moving the bob to the scratch mark on the rod, they possibly found the regulator nut almost half a turn away from the desired reading on the nut (the calibration setting), making it ambiguous whether to turn to the left or to the right to get the proper number on the nut. This changes the pendulum length by about one third of a millimetre in one or the other direction. This meant now and then the pendulum length was a full turn of the nut away from where it should be. I had a look at the observations of William Bayly, who was with Captain Furneaux on the Adventure at the time. I solved for his pendulum lengths and there also seem to be two clock rates which have been similarly affected, but now the other way, with the pendulum length 0.0258 inch too long instead of too short. It was pointed out to me that Wales' initials are on the relevant letters from Dawes (Morrison, 2008). I am

The Resolution, Captain Cook and William Wales														L Solved
Places	Date	Clock B gains on sidereal time		Latitude		Lat D.dd	Longitude		EGM 2008 normal gravity	Add Gravity Anomaly	clock rate sec	Obsv'd swing arc D.d		with obsvd arc from vert
Greenwich	March 1772	0	5.03	51	28.67N	51.48	0	0	9.81197137	0.00010	5.03	1.88		39.1308
Madeira	July 1772	0	-36.60	32	33.5N	32.56	17	11.25W	9.79525478	0.00015	-36.6	1.67		39.1031
Cape of Good Hope	Nov 1772	-1	15.43	33	55.25S	33.92	18	23.25E	9.79638135	0.00010	-75.43	1.63		39.1428
Dusky Bay	April 1773	0	4.07	45	47.5S	45.79	166	18E	9.80686841	0.00000	4.07	1.58		39.1124
Point Venus	August 1773	-1	28.42	17	29.5S	17.49	210	25.5E	9.78494686	0.00000	-88.42	1.65		39.1083
Queen Charlotte's Sound	Dec 1773	0	-21.12	41	6S	41.1	174	18.5E	9.80263555	0.00020	-21.12	1.63		39.1189
Point Venus	May 1774	-1	22.64	17	29.5S	17.49	210	25.5E	9.78494686	0.00000	-82.64	1.58		39.1034
Queen Charlotte's Sound	Oct 1774	0	-15.58	41	6S	41.1	174	18.5E	9.80263555	0.00020	-15.58	1.63		39.1139
Tierra del Fuego	Dec 1774	0	36.52	55	22S	55.37	70	1.33W	9.81534269	0.00015	36.52	1.63		39.1172
Cape of Good Hope	April 1775	0	-42.21	33	55.25	33.92	18	23.25	9.79638135	0.00010	-42.21	1.65		39.1125

The Resolution, Cook and Wales			
Places	Date	Adopted pendulum length	1775 Gravity value
Greenwich	March 1772	39.137	9.81364
Madeira	July 1772	39.111	9.79739
Cape of Good Hope	Nov 1772	39.137	9.79504
Dusky Bay	April 1773	39.111	9.80651
Point Venus	August 1773	39.111	9.78561
Queen Charlotte's Sound	Dec 1773	39.111	9.80086
Point Venus	May 1774	39.111	9.78684
Queen Charlotte's Sound	Oct 1774	39.111	9.80211
Tierra del Fuego	Dec 1774	39.111	9.81394
Cape of Good Hope	April 1775	39.111	9.79609

Table 2 (left). Pendulum lengths solved from the clock rates and pendulum amplitudes of the Shelton clock with Captain Cook on The Resolution. Table 3 (right). Gravity derived from correct pendulum lengths.

convinced William Wales was aware of the pendulum length error of William Dawes.

9. The importance of this for eighteenth century physical geodesy

Clairaut (1713 - 1765) had shown that the flattening of the earth could be derived from gravity measurements. The equation for the radius of an ellipsoid is:

$$r = a(1 - f \sin^2\Phi)$$

where a is the semi-major axis, f is the earth's flattening and Φ is the latitude.

This equation was important because gravity was known to be a function of the earth's radius. You could express or model gravity with almost the same equation, except with a plus sign, to show that gravity increases with latitude. So there is an inverse relationship with the radius value.

$$\gamma = \gamma_{equator}(1 + f^* \sin^2\Phi).$$ Clairaut's equation as in Magnizki (1960).

This is also known as Newton's gravity formula for normal gravity as in Torge (1980). The discerning reader will recognise that this looks like a first approximation of the modern international formulas for normal gravity, which I in fact used to come to the above conclusions about Dawes' measurement.

Clairaut showed that the earth's flattening f and the gravity flattening f^* could be worked out from each other, as in Heiskanen & Moritz (1967), p. 74, equation 2-99. This became known as Clairaut's theorem. The only other information you needed to know was the actual gravity at the equator $g_{equator}$ and the centrifugal force at the equator.

The meridian arc measurement of Peru of 1745 (Godin, La Condamine and Bouguer) and that of Lapland of 1736 (Maupertius and Clairaut) had originally resulted in an earth flattening estimate in the vicinity of 1/217 as shown by Jordan & Eggert (1948).

The gravity flattening is $f^* = (g_{pole} - g_{equator}) /g_{equator}$. In fact the gravity flattening derived from old gravity observations, together with Clairaut's theorem, suggests a flattening value for the earth in the 1/300 range and this could be what has suggested that one of the meridian arc measurements was in error. This was then corrected by Svanberg in 1810 by remeasuring part of the Lapland meridian arc, resulting in an early 19th century flattening estimate of 1/310 for the earth, which is closer to the true value.

Using highly precise pairs of gravity values at widely separated latitudes (Table 3), Nevil Maskelyne could solve for the gravity flattening f^* and the equatorial gravity $\gamma_{equator}$. Using *C*lairaut's theorem, one could then derive the earth's flattening from the gravity flattening. So these were interesting times. This is what precise gravity measurements like those from William Dawes contributed to.

10. Conclusion

It has been confirmed here that William Dawes observed gravity at Sydney Cove and the record has survived. But he made the same mistake in setting his pendulum regulator nut in 1788 as William Wales did between 1772 and 1775, an error which Wales described in the Introduction to his own and Bayly's observations. This effectively shortened both Wales' and Dawes' pendulum lengths by exactly a full turn of the nut and resulted in an

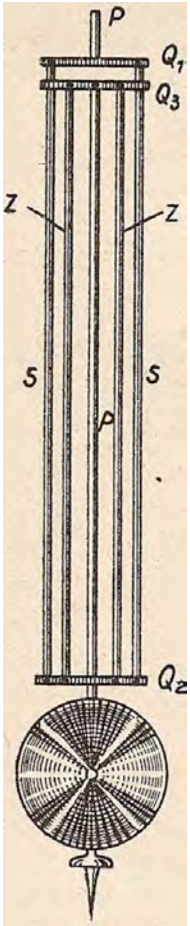


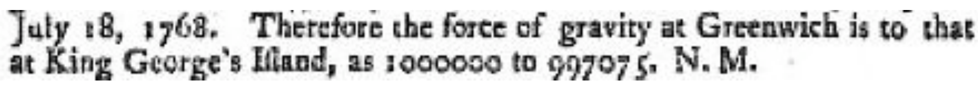
Fig. 4: Crossbar Q2 is suspended from Q1 by two steel bars (S). Crossbar Q2 moves lower when the steel bars (S) expand and lowers the pendulum bob, crossbar Q3 moves higher when the zinc bars (Z) expand and lifts the pendulum bob which is suspended from Q3. The right choice of lengths of the bars with different expansion coefficients creates temperature compensation. After Jordan & Eggert, (1948), Handbuch der Vermessungskunde (left).

Fig. 5: Sometimes the whole pendulum construction is suspended upside down as shown by the Shelton clock in the picture in the middle. After Armagh Observatory historical instruments.

Fig. 6: The picture at the right shows a regulator nut of an Earnshaw pendulum. After Armagh Observatory historical instruments.

estimated pendulum length of 39.111 inches instead of 39.137. In summary, William Dawes' September 1788 observation of the acceleration of gravity resulted in g = 9.79705 m/s^2, or **979.705 gal** for Sydney Cove. This agrees well with modern absolute determinations and had an accuracy of about 25 parts per million.

Acknowledgements: The research for this paper was carried out in support of Morrison & Barko (2009) who investigated the 1788 meeting of Joseph Lepaute d'Agelet and William Dawes.


July 18, 1768. Therefore the force of gravity at Greenwich is to that at King George's Iſland, as 1000000 to 997075. N. M.

Nevil Maskelyne's 6-figure relative gravity value, on page 405 of the 1771 Philosophical Transactions of the Royal Society.

REFERENCES

Bayly, W., 1782, *The original astronomical observations made in the course of a voyage to the Northern Pacific Ocean, for the discovery of a North East or North West passage wherein the north west coast of America and north east coast of Asia were explored in His Majesty's ships the Resolution and Discovery, in the years MDCCLXXVI, MDCCLXXVII, MDCCLXXVIII, MDCCLXXIX, and MDCCLXXX*, P. Elmsly and Mess. Mount and Page, Printed by William Richardson, London. http://www.canadiana.org/ECO/mtq?doc=17414

Giancoli, D.C., 1988, *Physics for Scientists and Engineers, with Modern Physics*, 2nd Ed., Prentice Hall, New Jersey.

Heiskanen,W. and Moritz, H., 1967, *Physical Geodesy*, W.H. Freeman and Company, San Francisco.

Howse, D. and Hutchinson, B., 1969, *The Saga of the Shelton Clocks*, Reprinted from Antiquarian Horology, Journal of the Antiquarian Horological Society, London.

Howse, D., 1969, *Captain Cook's Pendulum Clocks*, Reprinted from Antiquarian Horology, Journal of the Antiquarian Horological Society, London.

Jordan, W. and Eggert, O., 1948, *Handbuch der Vermessungskunde, Landvermessung, Sphär. Berechnungen und Astronomische Ortsbestimmung*, Metzlersche Verlags-buchshandlung, Stuttgart.

Laurie, P.S.,1988, *William Dawes, and Australia's first observatory*, Quarterly Journal of the Australian Astronomical Society, Vol 29, pp.469-482. http://articles.adsabs.harvard.edu//full/1988QJRAS..29..469L/0000469.000.html

Lomb, N., 2004, *Transit of Venus, the scientific event that led Captain Cook to Australia*, Powerhouse Publishing, Sydney.

Magnizki, W.A., Browar, W.W. and Schimbirew, B.P.,1964, *Theorie der Figur der Erde*, Verlag für Bauwesen, Berlin.

Morrison, D. and Barko, I., 2009, *Dagelet and Dawes: Their Meeting, Their Instruments and the First Scientific Experiments on Australian Soil*, Historical Records of Australian Science Vol 20, Nr 1, June 2009, CSIRO publishing for Australian Academy of Science. http://www.publish.csiro.au/nid/108/paper/HR09002.htm

Morrison, D., 2008, Correspondence with one of the authors of Morrison & Barko (2009), above.

Nicholson, J., 1825, *The Operative Mechanic and British Machinist*, Knight and Lacey, London.

Royal Society of Great Britain, 1771, *Philosophical Transactions, Giving Some Account of the Present Undertakings, Studies and Labours of the Ingenious, in Many Considerable Parts of the World, Vol. LXI, For the Year 1771*, Printed for Lockyer Davis, Printer to the Royal Society, London, 1772. (Google books.)

Torge, W., 1980, *Geodesy*, Walter de Gruyter, Berlin.

Vanicek, P., 1986, *Geodesy, The Concepts*, North Holland, Elsevier Science Publishers B.V., Amsterdam.

Wales, W. and Bayly, W., 1777, *The Original Astronomical Observations Made in the Course of a Voyage Towards the South Pole, and round the World in His Majesty's Ships the Resolution and Adventure, in the years MDCCLXXII, MDCCLXXIII, MDCCLXXIV, and MDCCLXXV*, J. Mount & T. Page, W & A Strahan, London.

Zupko, R.E., 1990, *Revolution in Measurement*, The American Philosophical Society, Philadelphia.

BIOGRAPHICAL NOTE

Case Bosloper is a retired Senior Surveyor of the Survey Infrastructure & Geodesy team of the former Department of Lands of NSW. Case is currently secretary of the AuScope GNSS Sub-Committee. He served for many years as the NSW representative on the Intergovernmental Committee on Surveying and Mapping, Geodesy Technical Sub-Committee (ICSM GTSC) and was one of the co-authors of the 1990 ICSM Standards and Specifications for Control Surveys. Case holds a 1971 master's degree in Geodesy from the Delft University of Technology (Geodetic Engineer) and a 1991 master of surveying science degree from UNSW.

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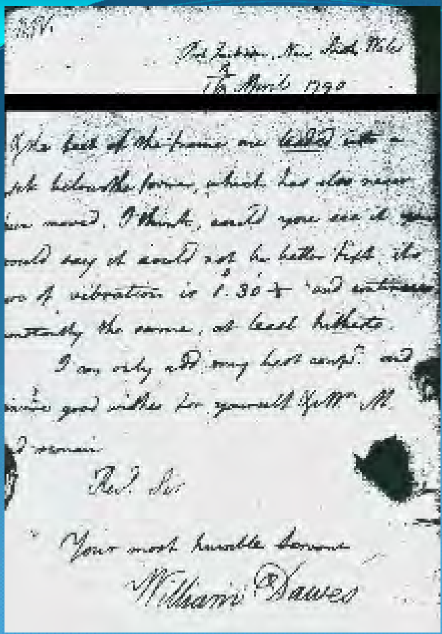
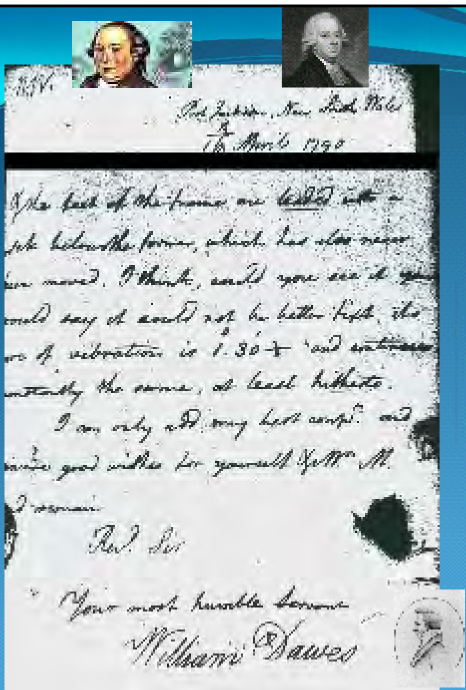


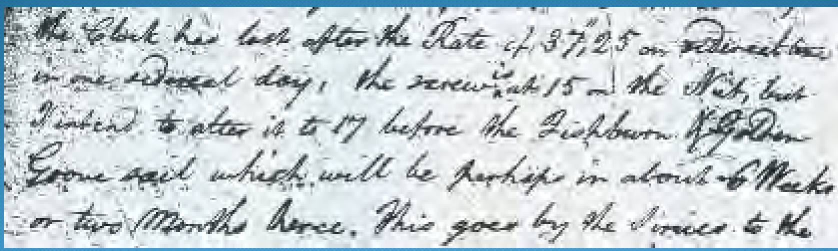
William Dawes' Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER, Australia



Clock Rate and setting of nut

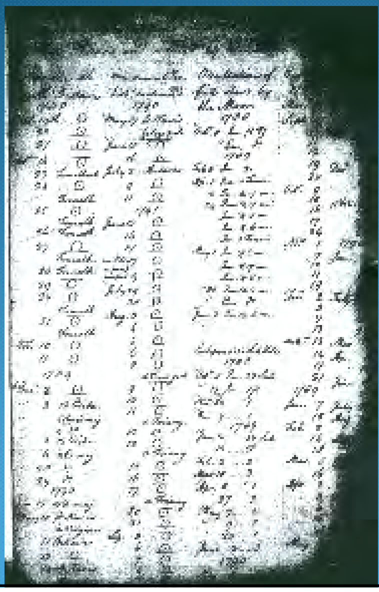
Oct 1, 1788

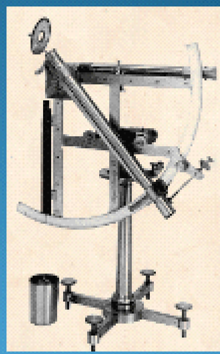

The Clock has lost after the Rate of 37,25 on sidereal time in one sidereal day, the screw is at 15 on the Nut, but I intend to alter it to 17 before the Fishburn & Golden Grove sail which will be perhaps in about 6 Weeks or two Months hence. This goes by the Sirius to the

William Dawes

1762 - 1836

Dawes' List of Observations





Gravity Observation by Cook

Equal Altitudes for the Going of the Clock N° 2, when at St. Peter and Paul the fecond time.

1779.	Time of Noon per Clock uncorrect. H. ' ''	Half Interval of Obfervations. H. ' ''	Time of Noon per Clock correct. H. ' ''	Clock faft for Sidereal Time. ' ''	Daily Rate of the Clock. ''	[illegible]	Phenomena and Remarks.
♃ Aug. 26.	10 18 38, 1	4 58 1	10 19 1,56	0 48,06	gaining.	14	Sun.
☿ Sept. 1.	10 41 49, 1	4 16 54	10 42 10,70	2 1,2	12,18	25	Do.
♃ —— 2.	10 45 40, 0	4 5 2	10 46 0,29	2 13,79	12,59	20	Do.
♄ —— 4.	10 53 18, 0	5 0 0	10 53 42,68	2 41,83	14,02	16	Do.
☉ —— 5.	10 57 7, 6	4 54 42	10 57 31,90	2 54,40	12,57	16	Do.
☽ —— 6.	11 0 57,75	4 46 0	11 1 21,75	3 6,85	12,45	12	Do.
♂ —— 7.	11 4 48,18	4 51 12	11 5 12,84	3 20,54	13,69	17	Do.
☿ —— 8.	11 8 38,41	4 43 30	11 9 2,77	3 34,47	13,93	14	Do.
♃ —— 16.	11 39 27,59	3 45 48	11 39 50,35	5 36,05	15,07	12	Do.
☉ —— 19.	11 51 13, 0	3 9 20	11 51 22,62	6 22,37	15,44	11	Do.
♀ —— 24.	12 10 14,09	4 26 6	12 10 36,39	7 37,09	14,94	12	Do.
♄ —— 25.	12 14 4,97	3 30 48	12 14 27,83	7 52,00	14,91	20	Do.
☉ —— 26.	12 17 53,48	4 12 6	12 18 17,93	8 7,10	15,10	16	Do.
27.	12 21 42,74	4 2 7	12 22 9,69	8 21,86	14,76	12	Do.

The pendulum vibrated from 1° 31'¼ to 1° 32' on each fide (o), which is 7'' more than before; this feems owing to the weather being much warmer.

Philosophical Transactions of the Royal Society
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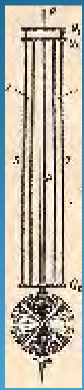
Astronomical Observations [made at …] for determining the going of the Clock

sent thither by the Royal Society

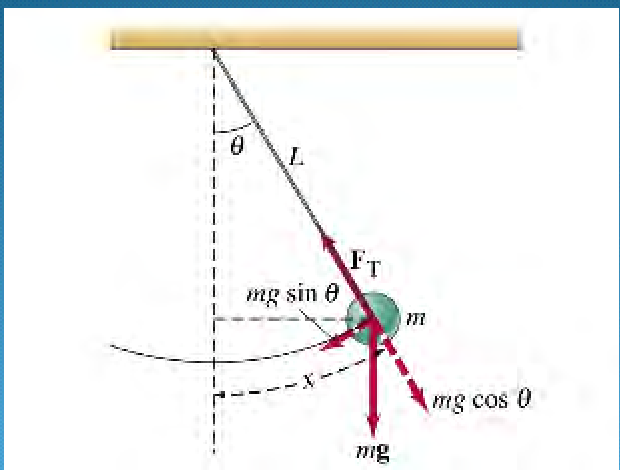
in order to find the Difference of Gravity between the Royal Observatory at Greenwich and the place where the Clock was set up [.....].

The Shelton Astronomical Regulator Clock

Error Budget



Linearity issue

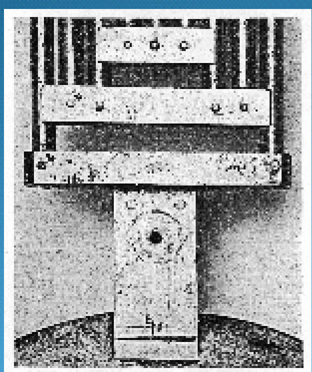


Pendulum equation

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{2^2}\sin^2\frac{\theta_M}{2} + \frac{1}{2^2}\frac{3^2}{4^2}\sin^4\frac{\theta_M}{2} + \cdots\right)$$

$$g = (4\pi^2 L/T^2) * (1 + \Delta)^2$$

Setting the Pendulum length



A Regulator Nut

Solve for Pendulum Lengths with EGM 2008

Mean 39.111₂ inches, Std dev of mean O.002 inch (51 microns)

The Resolution, Captain Cook and William Wales													L Solved
Places	Date	Clock B gains on sidereal time		Latitude		Lat D.dd	Longitude		EGM 2008 normal gravity	Add Gravity Anomaly	clock rate sec	Obsv'd swing arc D.d	with obsvd arc from vert
Greenwich	March 1772	0	5.03	51	28.67N	51.48	0	0	9.81197137	0.00010	5.03	1.88	39.1308
Madaira	July 1772	0	-36.60	32	33.5N	32.56	17	11.25W	9.79525478	0.00015	-36.6	1.67	39.1031
Cape of Good Hope	Nov 1772	-1	15.48	33	55.25S	33.92	18	23.25E	9.79638135	0.00010	-75.48	1.68	39.1428
Dusky Bay	April 1773	0	4.07	45	47.5S	45.79	166	18E	9.80586841	0.00000	4.07	1.58	39.1124
Point Venus	August 1773	-1	28.42	17	29.5S	17.49	210	25.5E	9.78494586	0.00000	-88.42	1.65	39.1083
Queen Charlotte's Sound	Dec 1773	0	-21.12	41	6S	41.1	174	18.5E	9.80263555	0.00020	-21.12	1.68	39.1180
Point Venus	May 1774	-1	22.64	17	29.5S	17.49	210	25.5E	9.78494586	0.00000	-82.64	1.58	39.1034
Queen Charlotte's Sound	Oct 1774	0	-15.58	41	6S	41.1	174	18.5E	9.80263555	0.00020	-15.58	1.63	39.1139
Tierra del Fuego	Dec 1774	0	36.52	55	22S	55.37	70	1.33W	9.81534269	0.00015	36.52	1.63	39.1172
Cape of Good Hope	April 1775	0	-42.21	33	55.25	33.92	18	23.25	9.79638135	0.00010	-42.21	1.65	39.1125

Length change for one rev of nut

From length ratio to length difference:

$$L_0/L_1 = T_0^2 / T_1^2$$

$$L_1 - L_0 = L_1 - (L_1 * T_0^2 / T_1^2)$$

$$= L_1(1 - T_0^2 / T_1^2)$$

Estimate L with g/π^2

Result: One revolution of nut is 0.0258 inch of length change for the pendulum

London pendulum length or not?

From normal gravity ratio to pendulum period ratio:

$$T_L^2 / T_S^2 = g_S / g_L$$

$$T_L = T_S \sqrt{(g_S / g_L)}$$

Conclusion: William Dawes pendulum is too short by one revolution of the regulator nut.

Result:

Sydney Cove gravity value in 1788

979.705 gal

The Appropriate Pendulum Lengths

The Resolution, Cook and Wales			
Places	Date	Adopted pendulum length	1775 Gravity value
Greenwich	March 1772	39.137	9.81364
Madeira	July 1772	39.111	9.79739
Cape of Good Hope	Nov 1772	39.137	9.79504
Dusky Bay	April 1773	39.111	9.80651
Point Venus	August 1773	39.111	9.78561
Queen Charlotte's Sound	Dec 1773	39.111	9.80086
Point Venus	May 1774	39.111	9.78684
Queen Charlotte's Sound	Oct 1774	39.111	9.80211
Tierra del Fuego	Dec 1774	39.111	9.81394
Cape of Good Hope	April 1775	39.111	9.79609

William Dawes' Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER, Australia

THE END

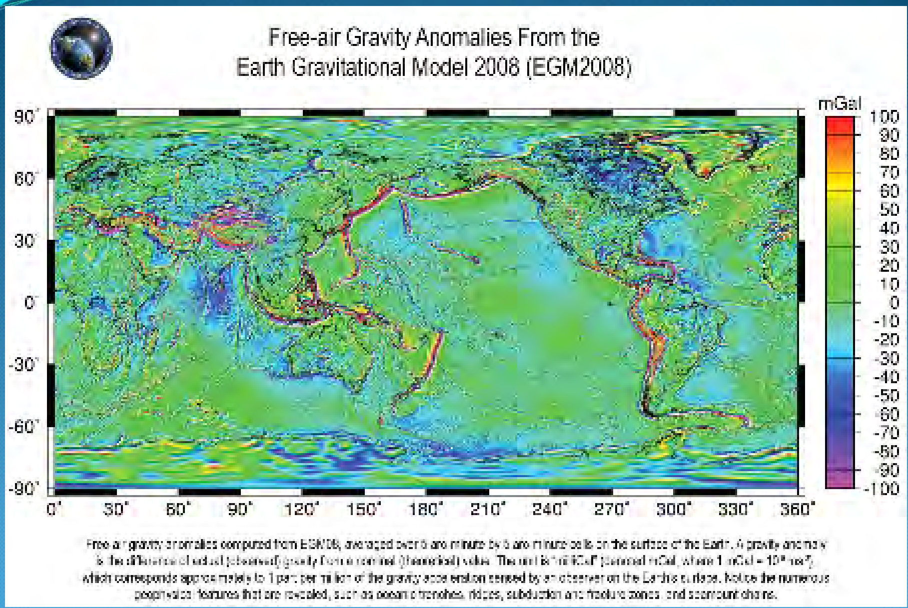
•Supporting background information

A Six-figure relative gravity value in 1768

Page 405, Philosophical Transactions of the Royal Society, 1771

July 18, 1768. Therefore the force of gravity at Greenwich is to that at King George's Island, as 1000000 to 997075. N. M.

EGM 2008 Anomalies



At equator

$$\gamma_{eq} = \frac{GM}{a^2}\left\{1 - 3J_2\left(\tfrac{1}{2}\right) - \frac{\omega^2 a^3}{GM}\right\}$$

$a = 6378137$

$GM = 3.986004418 \times 10^{14}$

$J_2 = 484.16514179 \times 10^{-6}$

$\sqrt{5}\, J_2 = 1082.626174$

$\omega = 7292115 \times 10^{-11}$ rad/sec.

$f = 1/298.257223563$

$$\gamma_{eq} = 9.798285479\{0.998162547\} = 9.780281594$$

$$\beta = -f + \frac{5}{2}m - \frac{17}{14}fm + \frac{15}{4}m^2$$

$$\beta_1 = \frac{1}{8}f^2 - \frac{5}{8}fm$$

$$m = \frac{\omega^2 a^2 b}{GM} = \frac{\omega^2 a^3(1-f)}{GM} = 3449.786507 \times 10^{-6}$$

Start with $f = 1/298.257223$

$$f = \frac{3}{2}J_2 + \frac{m}{2} + \frac{9}{8}J_2^2 + \frac{15}{28}J_2 m + \frac{3}{56}m^2$$

$$\gamma_0 = \gamma_{eq}\left(1 + \beta\sin^2\varphi + \beta_1\sin^2 2\varphi\right)$$

$$b = a(1-f)$$

Used J_2 as the not-normalized second degree zonal gravitational coefficient $(\sqrt{5}\,\bar{C}_{2,0})$

Iter. 1 $J_2 = \sqrt{5}\left(\bar{C}_{2,0}\right) = 1082.626174$

start $m_1 = 3449.786507 \times 10^{-6}$ $f_1 = 1/298.2572236$

$\frac{3}{2}J_2$	1623.939261	10^{-6}
$m/2$	1724.893254	10^{-6}
$9/8\, J_2^2$	1.318589	10^{-6}
$15/28\, J_2 m$	2.000801	10^{-6}
$3/56\, m^2$	0.637555	10^{-6}
	3352.789460	10^{-6}

$\longrightarrow f = 1/298.259110$

Recalculate $\rightarrow$ iteration 2: $m_2 = 3449.786581$

$\frac{3}{2}J_2$	1623.939261	10^{-6}
$m/2$	1724.893291	10^{-6}
$9/8\, J_2^2$	1.318589	10^{-6}
$15/28\, J_2 m$	2.000801	10^{-6}
$3/56\, m^2$	0.637555	10^{-6}
	3352.789497	10^{-6}

$\longrightarrow f = 1/298.2591066$

Iteration 3 $m_3 = 3449.786581$

Has converged.

$\longrightarrow$ f now stable, so m stable.

β	$-f$	$-$ 3352.789497	10^{-6}
	$\frac{5}{2}m$	$+$ 8624.466455	
	$-\frac{17}{14}fm$	$-$ 14.044925	
	$+\frac{15}{4}m^2$	$+$ 44.628353	
		5302.260387	10^{-6}

β_1 $\quad \frac{1}{8}f^2 \quad 1405149.676 \; 10^{-12}$

$-\frac{5}{8}fm \quad 7229005.140 \; 10^{-12}$

$-5823855.464 \; 10^{-12}$

or $5.8139 \; 10^{-6}$

OR

$$\gamma_0 = 9.7802816\,(1 + 0.0053022\,6\sin^2\varphi - 0.000005814\,\sin^2 2\varphi)\ \mathrm{ms^{-2}}$$

for EGM 2008

In six decimals (and 7 for coefficients):

$$\gamma_0 = 9.780282\,(1 + 0.0053023\sin^2\varphi - 0.0000058\sin^2 2\varphi)\ \text{in } \mathrm{ms^{-2}}$$

for WGS84 / EGM 2008.

Philosophical Transactions of the Royal Society

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[329.]

XLIII. *Astronomical Observations, made in the Forks of the River* Brandiwine *in* Pennsylvania, *for determining the going of a Clock sent thither by the* Royal Society, *in order to find the Difference of Gravity between the Royal Observatory at* Greenwich, *and the Place where the Clock was set up in* Pennsylvania; *to which are added, an Observation of the End of an Eclipse of the Moon, and some Immersions of* Jupiter's *First Satellite observed at the same Place in* Pennsylvania: *By* Charles Mason *and* Jeremiah Dixon.

Read December 15, 1768.