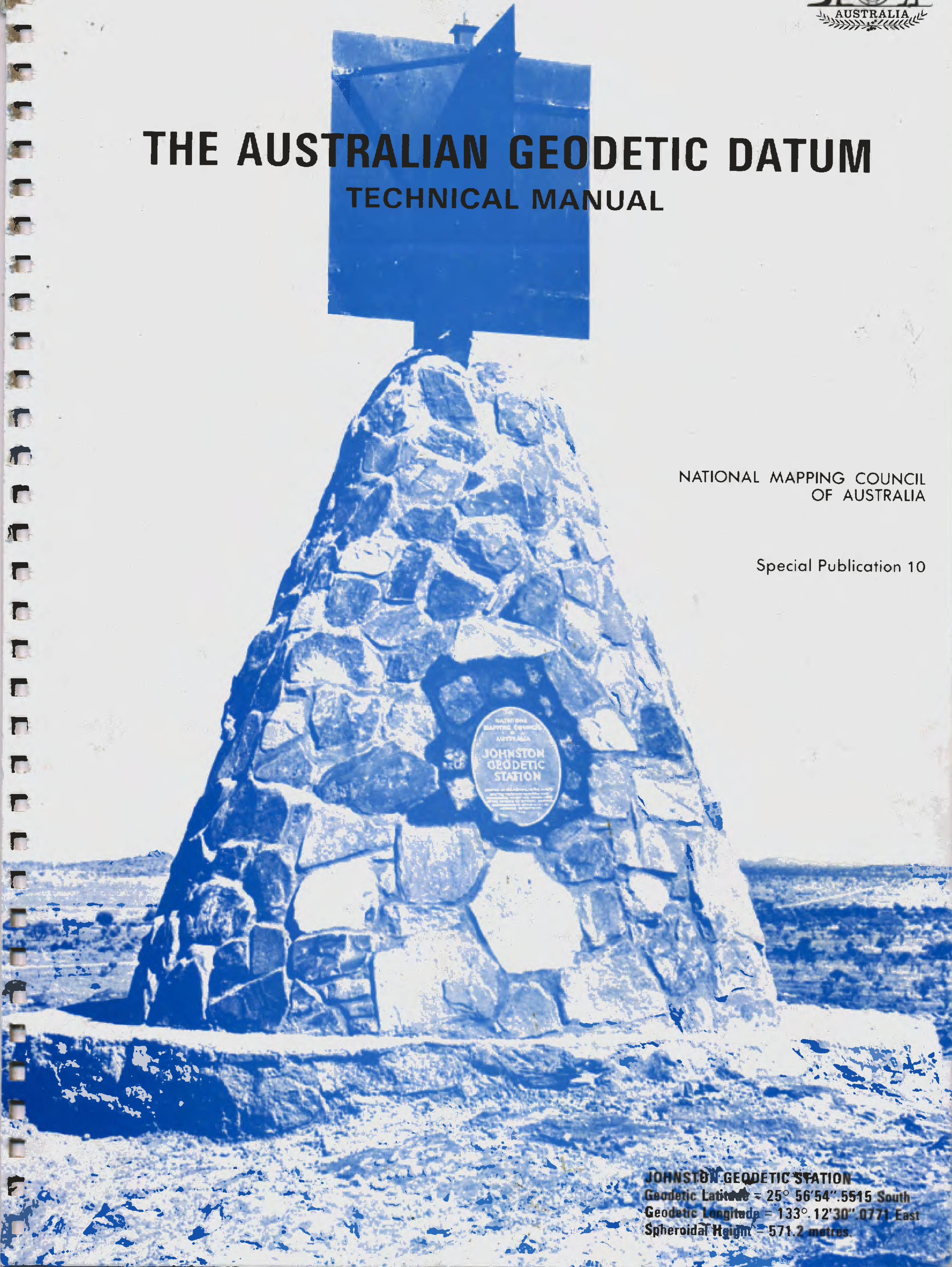
THE AUSTRALIAN GEODETIC DATUM

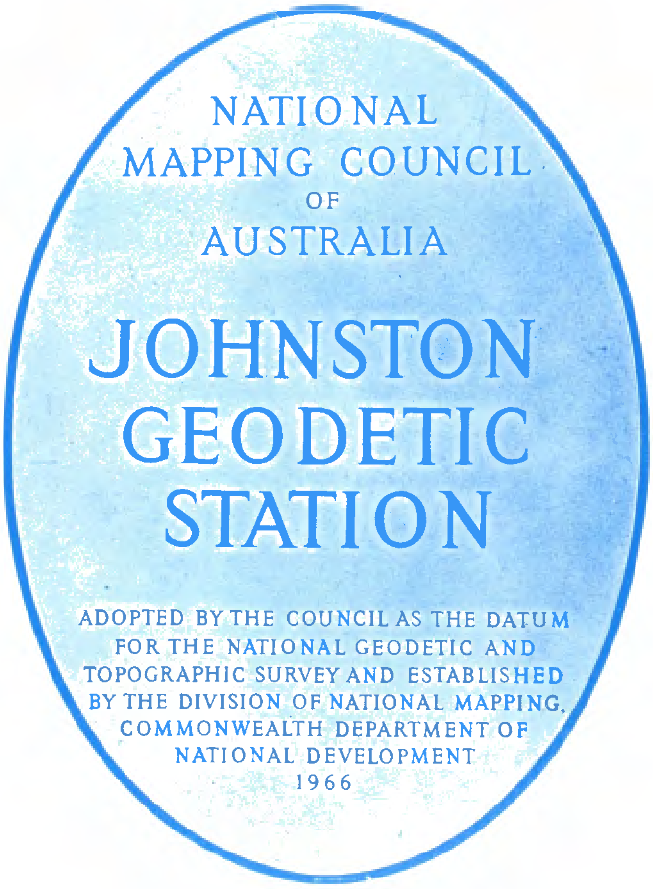
TECHNICAL MANUAL



NATIONAL MAPPING COUNCIL
OF AUSTRALIA

Special Publication 10

JOHNSTON GEODETIC STATION
Geodetic Latitude = 25° 56'54".5515 South
Geodetic Longitude = 133°.12'30".0771 East
Spheroidal Height = 571.2 metres.



The above wording appears on the inscription plate which is attached to the large stone cairn surmounting Johnston Geodetic Station. This station is located in the Northern Territory just north of the South Australian border and west of the Stuart Highway.

Johnston Geodetic Station cairn was erected towards the end of 1965 by officers of the Division of National Mapping, Department of National Development, and of the Lands and Survey Branch, Northern Territory Administration at Alice Springs in accordance with resolution No. 287 of the National Mapping Council of Australia which reads:

> The Council resolved that a special geodetic station be established and suitably monumented in the centre of Australia as the origin of the National Geodetic Survey and that this station be named Johnston in memory of Frederick Marshall Johnston, former Commonwealth Surveyor General and the first Director of National Mapping.

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FOREWORD

This manual replaces two earlier National Mapping Council publications: Special Publication 7—The Australian Map Grid Technical Manual, and Special Publication 8—The Australian Height Datum.

The first edition of Special Publication 7 was prepared in 1968 and a second edition was produced in 1972. Since the early 1970s, however, many technical advances have occurred in the field of computers, hand-held programmable calculators and in the application of space geodesy techniques to surveying and mapping.

Accordingly, at its fortieth meeting in 1982, the National Mapping Council established a Working Party to revise Special Publication 7 so that its contents would reflect these new technologies.

Subsequently, at its forty-first meeting in 1983, the Council decided to incorporate in the revised manual the contents of Special Publication 8 which was first published in 1979. As a result of this incorporation the title Special Publication 10—Australian Geodetic Datum Technical Manual was adopted.

The work contained herein reflects the dedicated efforts of the Working Party which was constituted as follows:

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The Working Party was assisted with contributions from Commonwealth and State mapping organisations and from institutions engaged in the teaching of surveying. Many useful suggestions received from these sources have been incorporated into this manual.

The National Mapping Council wishes to record its gratitude to all who have contributed to this fine manual and commend its use to those seeking a definitive explanation of the Australian Geodetic and Height Datums.

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Contents

1.

GENERAL NOTES

1.1 INTRODUCTION

1.1.1 Between 1858 and 1966, geodetic surveys in Australia were computed on either a State or regional basis using no fewer than four different spheroids and as many as twenty coordinate origins. Some of the larger States employed two or more origins simultaneously and various values were adopted for the imperial system units of length then in use.

1.1.2 The National Mapping Council, at its twenty-third meeting in April 1965, adopted the spheroid then recommended for general use by the International Astronomical Union. The Council decided to call this spheroid the Australian National Spheroid. Re-computation and adjustment of all geodetic surveys in Australia on this new spheroid were commenced by the Division of National Mapping in June 1965.

1.1.3 On 8 March 1966, all geodetic surveys in Australia and what is now Papua New Guinea were finally re-computed and adjusted on the then newly defined Australian Geodetic Datum. This datum was subsequently adopted by the National Mapping Council on 21 April 1966, during its twenty-fourth meeting in Melbourne. The Australian Geodetic Datum was proclaimed in the *Commonwealth Gazette* No. 84 of 6 October 1966.

1.1.4 The introduction of the Australian Geodetic Datum created a unique system of coordinates for geodetic surveys all over Australia that was free from the discontinuities caused by the use of various State coordinate origins. Coordinates on the Australian Geodetic Datum provide a firm foundation on which lower order surveys and all mapping can be based; furthermore, they provide a basis for a point reference system on which any point in Australia—for example, an oil well, a mineral deposit, a civil defence headquarters, or a bushfire outbreak—can be described in precise and unambiguous terms.

1.1.5 Geodetic coordinates are usually computed in latitude and longitude. For many purposes, including mapping, a system of rectangular grid coordinates—eastings and northings—is more convenient. Prior to 1966, a Transverse Mercator projection, based on the imperial system with measurements in yards, had long been used in Australia for this purpose. With the introduction of the Australian Geodetic Datum occurring during a period of gradual change to the metric system, the opportunity was taken to change to the Universal Transverse Mercator Grid with measurements in metres. This new grid was called the *Australian Map Grid* (AMG).

1.1.6 The aims of this manual are:

- to define the Australian Geodetic Datum and the Australian Map Grid;
- to define the World Geodetic System 1972 and its use as a geodetic datum in conjunction with the Universal Transverse Mercator Grid system for all mapping of Australian offshore islands and external territories lying outside the limits of the Australian Map Grid;
- to define standard symbols, terms and sign conventions for use throughout Australia and its offshore islands and external territories;
- to provide information on the reduction of slope distances to the spheroid;
- to provide a set of numerical examples, using the full rigour of the defining formulae, as a standard against which computer and programmable calculator programs can be judged;
- to set out each numerical example in such a way so as to facilitate the construction of a logic diagram from which it would then be possible to write a computer program in any given language for any given computer or programmable calculator;
- to show how to adapt certain formulae for simplified computations;
- to show how to use grid references;
- to define the Australian Height Datum;
- to give information on geoid-spheroid separation for the Australian National Spheroid.

1.2 THE AUSTRALIAN GEODETIC DATUM (AGD)

1.2.1 Geodetic surveys in Australia are computed on the Australian National Spheroid for which the defining parameters are:

Major semi-axis,	a	=	6 378 160 metres
Flattening,	f	=	1/298·25 exactly

Derived functions for this spheroid are given in paragraph *3.2*.

1.2.2 In 1966, the minor axis of the spheroid was defined as being parallel to the earth's mean axis of rotation at the start of 1962. In 1970 the National Mapping Council decided to adopt the Conventional International Origin (CIO, 1967), previously known as the mean pole 1900·0—1906·0, for the direction of the minor axis. The Council decided that no change in the 1966 coordinates was necessary. The Australian Geodetic Datum reference meridian plane of zero longitude is defined as being parallel to the Bureau International de l'Heure (BIH) mean meridian plane near Greenwich. This in turn gives a value of 149°00′ 18″·885 East for the plane contained by the vertical through the Mount Stromlo photo zenith tube and the CIO. The position of the centre of the spheroid is defined by the following coordinates of *Johnston Geodetic Station*:

Geodetic Latitude	=	25°56′54″·551 5 South
Geodetic Longitude	=	133°12′30″·077 1 East
Spheroidal Height	=	571·2 metres

1.2.3 The size, shape, position and orientation of the spheroid were thus completely defined, and together defined the Australian Geodetic Datum. The coordinates for Johnston Geodetic Station were derived from astronomical observations at 275 stations on the geodetic survey distributed all over Australia. The spheroidal height was adopted to be 571·2 metres, which is equal to the height of the station above the geoid as computed by trigonometrical levelling in 1965.

1.2.4 Due to the almost complete lack of geoidal profiles at the time of the 1966 national adjustment, it was then assumed that geoid-spheroid separation was zero not only at Johnston Geodetic Station but also at all other geodetic stations listed in this adjustment. This assumption implied that every distance used in the 1966 adjustment, although theoretically a spheroidal distance, was actually a geoidal or sea level distance.

All computations relating to the 1966 national adjustment, including those referred to in this manual, assume zero geoid-spheroid separation.

Since the 1966 national adjustment, much information has become available on geoid-spheroid separation at Johnston Geodetic Station and in other areas of Australia. A general summary of this information is given in Chapter 9.

1.2.5 Since 1966, there have been several readjustments of the national geodetic survey. Each readjustment has been referred to as a Geodetic Model of Australia (GMA) and is identified by the year in which the data set used in the readjustment was compiled. The latest adjustment, GMA82, has used the most recent observations available, including satellite Doppler, laser ranging to satellites and Very Long Baseline Interferometry (VLBI) observations.

Recognising the need for Australia to eventually convert to a geocentric geodetic datum, the National Mapping Council, at its forty-second meeting in October 1984, resolved that the GMA82 adjustment would be adopted as the first step in the conversion process. However, the Council also resolved that members could use their discretion in the timing of the conversion process.

The GMA82 adjustment maintained the Australian Geodetic Datum as originally defined by the combination of the 1966 coordinate set for Johnston Geodetic Station and the defining parameters for the Australian National Spheroid.

In order to forestall any confusion arising with regard to the terminology to be used in conjunction with the 1966 and GMA82 adjustments, the National Mapping Council has adopted the following definitions for general usage:

- Datum — Australian Geodetic Datum
- Spheroid — Australian National Spheroid
- The 1966 Coordinate Set — AGD66 (geographical coordinates)
— AMG66 (grid coordinates)
- The 1982 Adjustment/Least Squares Solution — GMA82
- The 1984 Adopted Coordinate Set — AGD84 (geographical coordinates)
— AMG84 (grid coordinates)

Unlike the 1966 adjustment, the GMA82 adjustment is a truly spheroidal adjustment. Therefore, any observations used in conjunction with the AGD84 coordinate set should first be reduced to the Australian National Spheroid using the appropriate geoid-spheroid separation values in terms of N = +4·9 metres at Johnston Geodetic Station.

1.3 THE AUSTRALIAN MAP GRID (AMG)

1.3.1 Coordinates on the Australian Map Grid are derived from a Transverse Mercator projection of latitudes and longitudes on the Australian Geodetic Datum. The coordinates are defined by the formulae for easting and northing given in paragraph *4.5* first published by J.C.B. Redfearn in the *Empire Survey Review*, No. 69, 1948. They are correct to less than 1mm anywhere in a given grid zone. For the purposes of this definition, these formulae, and the formulae for meridian distance given in paragraphs *4.2* and *4.3* can be regarded as exact, and not as the opening terms of an infinite series.

1.3.2 The Australian Map Grid corresponds with the Universal Transverse Mercator Grid, as follows:

- coordinates are in metres;
- zones are 6° wide plus overlapping belts of 80 kilometres at each grid junction;

- AMG zones are numbered from zone 49 with central meridian 111°E to zone 57 with central meridian 159°E;
- the origin of each zone is the intersection of the central meridian with the equator;
- a central scale factor, k_0, is defined as 0·999 6;
- eastings E, are defined by adding 500 000 metres to the value of E′, given by the formula in paragraph *4.5.1*;
- in the southern hemisphere, northings N, are defined by adding 10 000 000 metres to the negative value of N′ given by the formula in paragraph *4.5.2*.

Diagrams illustrating the basic characteristics of the Universal Transverse Mercator projection and the concepts of zone width, zone overlap, central meridian, projection plane and geoid-spheroid separation are shown in Figures 2.1 and 2.2 respectively.

1.4 LIMITS OF THE AUSTRALIAN MAP GRID

The Universal Transverse Mercator Grid system is of world-wide application, but the geodetic reference system in local use varies between regions, so the limits within which the Australian Map Grid is to be used need to be defined. The Australian Map Grid covers the Australian mainland, Tasmania and features close to their shores (see Annex A). The grid does not cover Lord Howe, Macquarie, Cocos, Christmas, Norfolk, Heard and McDonald Islands; neither does it cover the Australian Antarctic Territory.

1.5 GEODETIC DATUM FOR AUSTRALIAN OFFSHORE ISLANDS AND EXTERNAL TERRITORIES

1.5.1 At its thirty-eighth meeting, held in September 1980, the National Mapping Council adopted the United States Department of Defense World Geodetic System 1972 (WGS 72) as the geodetic datum for all mapping of Australian offshore islands and external territories lying outside the limits of the Australian Map Grid.

1.5.2 The Australian National Spheroid and the WGS 72 spheroid differ in size, shape and orientation with a resulting vector difference of approximately 195 metres between their respective centres. Due to these dissimilarities, the vector difference between the related coordinate reference systems varies over the area in which the Australian National Spheroid is to be used, but is generally of the order of 200 metres. Maps and charts published for the same area at the same scale and using the Transverse Mercator projection, but compiled on the respective geodetic datums, will exhibit this vector difference as a shift between the map positions of identical topographic or bathymetric features when corresponding graticule lines are superimposed.

1.6 THE WORLD GEODETIC SYSTEM 1972 (WGS 72) DATUM

1.6.1 A world geodetic system can be defined as one in which all points of that system are fixed in relation to the earth's centre of mass. A practical addendum to this definition is usually the inclusion of the parameters of an earth spheroid which best fits the geoid as a whole.

1.6.2 The WGS 72 datum is but one of several geocentric (earth-centred) coordinate reference systems currently in international use. This datum was derived from a large scale least squares adjustment of a mass of data gathered from global Doppler and optical satellite observations, surface gravity surveys, continental triangulation and trilateration surveys, transcontinental high precision traverses, precise astronomic observations and space probes. This adjustment resulted in the fundamental geodetic parameters required to define the WGS 72 mean earth spheroid which are as follows:

Major semi-axis, $a = 6\,378\,135$ metres
Flattening, $f = 1/298{\cdot}26$

Derived functions for this spheroid are given in paragraph *3.2.1*.

1.6.3 In the derivation of the WGS 72 spheroid, the direction of the minor axis was defined to be parallel to the vertical through the CIO. The plane of zero geodetic longitude was defined to be parallel to the plane contained by the vertical through the BIH mean observatory near Greenwich and the CIO. Both the BIH mean observatory and the CIO are computed points defined by the adopted latitudes and longitudes of participating observatories.

1.6.4 The position, size, shape and orientation of the WGS 72 spheroid are thus completely defined and, together with an adopted geopotential model for the earth's gravity field and certain other constants, define the World Geodetic System 1972.

1.7 USE OF THE UNIVERSAL TRANSVERSE MERCATOR GRID SYSTEM IN CONJUNCTION WITH THE WORLD GEODETIC SYSTEM 1972 DATUM

1.7.1 In adopting the WGS 72 datum for all mapping of Australian offshore islands and external territories lying outside the limits of the Australian Map Grid, the National Mapping Council also decided that rectangular coordinates shall be obtained from the WGS 72 coordinates by the UTM projection north of latitude 80° South; and by the polar stereographic grid system south of latitude 79° 30′ South.

1.7.2 Coordinates on the UTM Grid are derived from a Transverse Mercator projection of latitudes and longitudes on the WGS 72 datum by means of Redfearn's formulae given in paragraph *4.5*. For the purposes of this definition, these formulae, and the formulae for the computation of meridian distance given in paragraphs *4.2* and *4.3*, can be regarded as exact.

1.7.3 The characteristics of the UTM grid system are identical with those already given for the AMG in paragraph *1.3*. Grid zone numbers correspond with the AMG and are numbered eastwards from zone 1 with central meridian 177° West to zone 60 with central meridian 177° East.

1.7.4 The derivation of rectangular grid coordinates from the WGS 72 coordinates by the polar stereographic projection south of latitude 79° 30′ South is beyond the scope of this manual. A full description of this projection can be found in *Map Projections Used By The U.S. Geological Survey, Geological Survey Bulletin 1532*, published by the U.S. Government Printing Office, Washington, 1982.

1.8 NUMERICAL EXAMPLES

1.8.1 The aim of the numerical examples included in this manual is to provide a standard against which computer and programmable calculator programs can be tested and compared.

1.8.2 In order to prove the various formulae, two test lines have been selected. One of these lines is located within the limits of the AMG where the formulae have been rigorously applied in computations on the Australian National Spheroid and the AMG. The other test line is located outside the limits of the AMG, and in the vicinity of the Australian Territory of Norfolk Island, where the formulae have again been rigorously applied in computations on the WGS 72 spheroid and the UTM Grid. Both test lines provide a comprehensive, though not extreme, test for all the formulae.

1.8.3 The test line located within the limits of the AMG lies in Victoria and connects trigonometrical stations Flinders Peak and Buninyong. This line is nearly 55 kilometres long in an azimuth of 307°, and runs across the boundary between AMG zones 54 and 55.

1.8.4 The test line located outside the limits of the AMG has one of its terminals coinciding with trigonometrical station "M" on Norfolk Island. The other terminal is a completely fictitious trigonometrical station, designated "X", lying nearly 57 kilometres in an azimuth of 69° 37′50″ from station "M". This line runs across the boundary between UTM Grid zones 58 and 59.

1.8.5 The numerical examples are given in full and a summary of the geodetic values for both test lines, as computed by Robbins's and Redfearn's Formulae, is given in Annex H. These examples have been computed using 64 bit precision, equivalent to 16 significant digits.

1.9 COMPUTER PROGRAMS

1.9.1 The computations described in this manual have been programmed for computers by various surveying and mapping authorities. Surveyors wishing to make use of these programs, or to obtain program listings and other related information, should contact the appropriate authority listed in Annex F.

1.9.2 The rigorous formulae given in Chapters 3 and 4 have been programmed for computers by the Division of National Mapping as follows:

ROBBINS — Latitude and Longitude from Distance and Azimuth or Distance and Azimuth from Latitude and Longitude, using Robbins's Formulae. See paragraphs *3.3* and *3.4*.

REDFEARN — Transformation of Geographic Coordinates to Grid or Grid Coordinates to Geographic, using Redfearn's Formulae. These formulae can be used in conjunction with the UTM Grid on any spheroid. See paragraphs *4.5* and *4.8*.

In addition, the following programs are available:

RAINSFORD — Latitude and Longitude from Distance and Azimuth or Distance and Azimuth from Latitude and Longitude using Rainsford's Formulae. These formulae are more appropriate than Robbins's Formulae for computation of very long lines on the spheroid.

LAUF — Conformal Transformation from Grid to Grid by Lauf's Formulae.

VARYCORD — A least squares adjustment of horizontal control surveys which is computed on the spheroid and gives both AGD and AMG values.

GANET — A least squares adjustment of horizontal control surveys which will accept positional observations, such as satellite Doppler fixes. This program has now superseded VARYCORD.

1.10 PROGRAMMABLE CALCULATOR SUPPLEMENT

The supplement to this manual contains programs written for the Hewlett Packard HP41-CV/CX programmable calculators relating to the various types of computations set out in Chapters 3, 4 and 5. This integrated set of programs is primarily intended for verification of field observations. In order to operate within the available precision and to conserve memory space, some of the rigorous formulae, such as those of Robbins, have been replaced and/or rationalised. Full documentation is supplied with the listings.

This supplement will be produced in a loose-leaf, low-cost form and it is proposed that it will be updated from time to time in order that the programs therein reflect the latest trends in programmable calculator technology.

2.

SYMBOLS, DEFINITIONS, SIGN CONVENTIONS AND REDUCTION OF MEASURED DISTANCES TO THE SPHEROID AND SEA LEVEL

2.1 SYMBOLS

The symbols used with the Australian National Spheroid and the Australian Map Grid, and also with the WGS 72 spheroid and the UTM grid as the latter apply to Australian offshore islands and external territories, are listed below. Many terms are more fully defined in paragraph *2.2* and are further illustrated in Figures 2.1, 2.2 and 2.3. Further symbols used with the Australian National Spheroid and the WGS 72 spheroid are listed in paragraph *3.2*.

THE GREEK ALPHABET

Alpha	A	α
Beta	B	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	E	ϵ
Zeta	Z	ζ
Eta	H	η
Theta	Θ	θ
Iota	I	ι
Kappa	K	κ
Lambda	Λ	λ
Mu	M	μ
Nu	N	ν
Xi	Ξ	ξ
Omicron	O	o
Pi	Π	π
Rho	P	ρ
Sigma	Σ	σ
Tau	T	τ
Upsilon	Y	υ
Phi	Φ	ϕ
Chi	X	χ
Psi	Ψ	ψ
Omega	Ω	ω

ϕ	=	Geodetic latitude, negative south of the equator.
ϕ_1,ϕ_2	=	Latitude at points 1 and 2 respectively.
ϕ_m	=	$(\phi_1+\phi_2)/2$ } and similarly for other terms.
$\Delta\phi$	=	$\phi_2-\phi_1$ } and similarly for other terms.
$\Delta\phi''$	=	$\Delta\phi$ expressed in seconds of arc. } and similarly for other terms.
λ	=	Geodetic longitude measured from Greenwich, positive eastwards.
λ_0	=	Geodetic longitude of a central meridian.
ω	=	Geodetic longitude measured from a central meridian, positive eastwards: $\omega = \lambda-\lambda_0$
E′	=	Easting measured from a Central Meridian, positive eastwards.
N′	=	Northing measured from the equator, negative southwards.
E	=	E′ + 500 000 metres.
N	=	N′ in the northern hemisphere.
	=	N′ + 10 000 000 metres in the southern hemisphere.
ρ,ν	=	Radii of curvature of the spheroid in meridian and prime vertical respectively.
α	=	Azimuth, clockwise through 360° from true north.
β	=	Grid bearing, clockwise through 360° from grid north.
θ	=	Plane bearing, clockwise through 360° from grid north.
γ	=	Grid convergence, positive when grid north is west of true north, negative when grid north is east of true north: $\beta = \alpha + \gamma$
δ	=	Arc-to-chord correction with sign defined by the equations: $\theta = \beta + \delta = \alpha + \gamma + \delta$
$\Delta\alpha$	=	Meridian convergence.
$\Delta\beta$	=	Line curvature.
s	=	Spheroidal distance.
s′	=	Sea level, or geoidal, distance.
S	=	Grid distance.
L	=	Plane distance.
m	=	Meridian distance, true distance from the equator, negative southwards.
G	=	Mean length of an arc of one degree of the meridian.

σ = m/G
a,b = Major and minor semi-axes of the spheroid.
f = $(a - b)/a$ = flattening.
e^2 = $(a^2 - b^2)/a^2$ = (eccentricity)^2
e'^2 = $(a^2 - b^2)/b^2$ = (second eccentricity)^2
n = $(a - b)/(a + b) = f/(2 - f)$
k_0 = Central scale factor = 0·999 6
k = Point scale factor.
K = Line scale factor.
t = $\tan \phi$
ψ = ν/ρ
ϕ' = Latitude for which $m = N'/k_0$ = Foot-point latitude.
t', ψ', ρ', ν' are functions of the latitude ϕ'.
R^2 = $\rho\nu$
R_α = Radius of curvature in a given azimuth.
r^2 = $R^2 k_0^2 = \rho\nu k_0^2$
r_m^2 = $\rho\nu k_0^2$ at ϕ_m.
h = Spheroidal height.
H = Height of a point above the geoid, or orthometric height.
N = h - H = geoid-spheroid separation.

Note: E′, N′, E, N, S, L, r, k and K include the central scale factor, k_0.
s, ρ, ν, R and m are true distances, which must be specifically multiplied by k_0 when necessary.

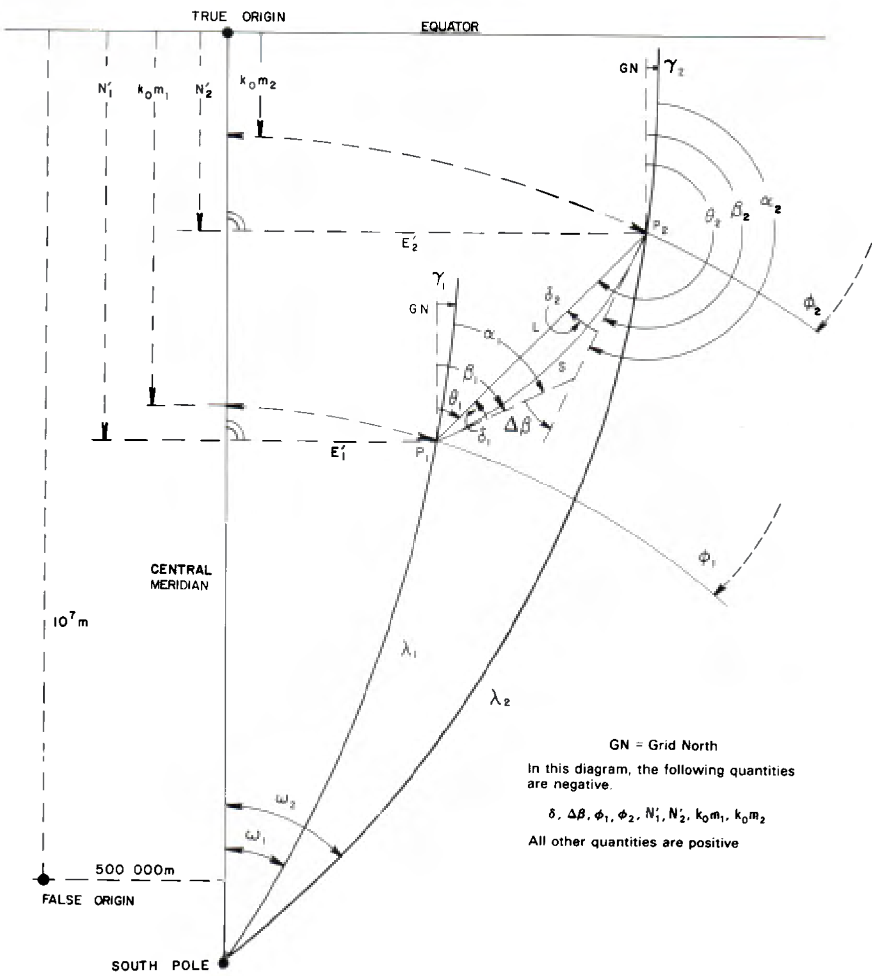


Figure 2.1 BASIC CHARACTERISTICS OF THE UNIVERSAL TRANSVERSE MERCATOR PROJECTION

2.2 DEFINITIONS—see also Figures 2.1, 2.2 and 2.3

2.2.1 AZIMUTH, MERIDIAN CONVERGENCE, SPHEROIDAL AND SEA LEVEL DISTANCES

Azimuth, α, is a horizontal angle measured from the spheroidal meridian clockwise from north through 360°.

Meridian Convergence, $\Delta\alpha$, is the change in the azimuth of a geodesic between two points on the spheroid:

$$\text{Reverse Azimuth} = \text{Forward Azimuth} + \text{Meridian Convergence} \pm 180°$$

$$\alpha_2 = \alpha_1 + \Delta\alpha \pm 180°$$

Spheroidal Distance, s, is the distance on the spheroid along either a normal section or a geodesic. The difference between the two is usually negligible, amounting to less than 20 millimetres in 3 000 kilometres.

Sea level, or geoidal, distance s'. When heights above the geoid, which are often referred to as orthometric heights, are used in the reduction of distances measured between points on the earth's surface to the surface of the geoid, the resulting distance is termed a sea level, or geoidal, distance s'. See paragraph *1.2.4.*

2.2.2 GRID BEARING, LINE CURVATURE AND GRID DISTANCE

A line on the spheroid of length s is projected on the grid as an arc.

Grid Bearing, β, at a point on the arc, is the angle between grid north and the tangent to the arc at the point. It is measured from grid north clockwise through 360°.

Line Curvature, $\Delta\beta$, is the change in grid bearing between two points on the arc.

$$\text{Reverse grid bearing} = \text{Forward grid bearing} + \text{Line curvature} \pm 180°$$

$$\beta_2 = \beta_1 + \Delta\beta \pm 180°$$

Grid Distance, S, is the length measured along the arc of the projected line whose spheroidal distance is s.

2.2.3 PLANE BEARING AND PLANE DISTANCE

A straight line can be drawn on the grid between the ends of the arc defined in paragraph *2.2.2.*

Plane Bearing, θ, is the angle between grid north and this straight line.

Plane Distance, L, is the length of this straight line. The difference in length between the plane distance, L, and the grid distance, S, is nearly always negligible.

Using plane bearings and plane distances, the formulae of plane trigonometry hold with complete rigour:

$$\tan\theta = \Delta E/\Delta N; \qquad \Delta E = L\sin\theta; \qquad \Delta N = L\cos\theta.$$

2.2.4 GRID CONVERGENCE

Grid convergence, γ, is the angular quantity to be added algebraically to an azimuth to obtain a grid bearing:

$$\text{Grid Bearing} = \text{Azimuth} + \text{Grid Convergence}$$

$$\beta = \alpha + \gamma$$

The sign of the grid convergence is determined by the formulae in paragraphs *4.3.3* and *4.7.3.* In the southern hemisphere, grid convergence is positive for points east of a central meridan, and negative west.

2.2.5 ARC-TO-CHORD CORRECTION

The *Arc-to-chord Correction,* δ, is the angular quantity to be added algebraically to a grid bearing to obtain a plane bearing:

$$\theta = \beta + \delta = \alpha + \gamma + \delta$$

The arc-to-chord corrections differ in amount and sign at the two ends of a line. The sign is determined by the formulae in paragraphs *5.3, 5.6* and *5.11.* Lines which do not cross the central meridian always bow away from the central meridian. In the rare case of a line which crosses the central meridian less than one-third of its length from one end, the bow is determined by the longer part. Refer to paragraph *5.12.4* for further explanation. Note that

$$\Delta\beta = \delta_1 - \delta_2$$

The arc-to-chord correction is sometimes called the 't-T' correction.

2.2.6 SCALE FACTORS

Point Scale Factor, k, is the ratio of an infinitesimal distance at a point on the grid to the corresponding distance on the spheroid:

$$k = dL/ds = dS/ds$$

It is the distinguishing feature of conformal projections, such as the Universal Transverse Mercator, that this ratio is independent of the azimuth of the infinitesimal distance.

Line Scale Factor, K. From point to point along a line on the grid the point scale factor will in general vary. The line scale factor is the ratio of a plane distance, L, on the grid to the corresponding spheroidal distance, s:

$$K = L/s \doteq S/s$$

Height Factor is the ratio of an infinitesimal horizontal distance at a point on the spheroid to the corresponding distance at a particular height above (or below) the spheroid.

Combined Point Scale and Height Factor is the product of the point scale factor and the height factor. It may be used to convert measured horizontal distances to grid distances. Its use is not rigorous, because the point scale factor is an approximation for the line or locality being computed.

2.2.7 HEIGHTS

Spheroidal Height, h, is the distance of a point above the spheroid measured along the normal from that point to the surface of the spheroid .

Geoidal Height, H, is the distance of a point above the geoid measured along the normal from that point to the surface of the geoid. It is also referred to as the orthometric height.

Geoid-Spheroid Separation, N, is the distance from the surface of the spheroid to the surface of the geoid measured along the normal to the spheroid. This distance can be either positive or negative depending upon whether the geoid is respectively above or below the spheroid.

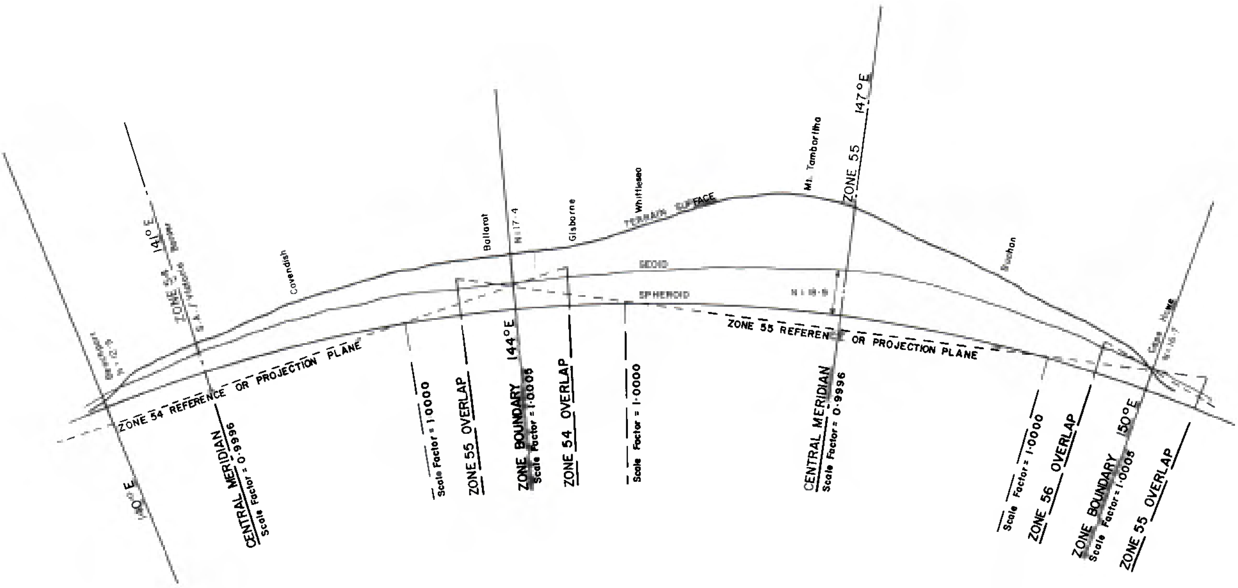


Figure 2.2 Section along latitude 37°30′ south through Victoria showing AMG Zones and relationship of AMG projection planes to the geoid, spheroid and terrain surface.

For location of cross section see Annex E

N values are shown in terms of N = +4·9 metres at Johnston Geodetic Station

NOT TO SCALE

2.3 REDUCTION OF MEASURED DISTANCES TO THE SPHEROID

2.3.1 Modern geodetic measurements are undertaken with electronic distance measurement equipment (EDM) between points on the earth's surface at different elevations above the surface of the spheroid. Due to the effects of atmospheric refraction, the light waves or microwaves used by EDM follow a curved path. Before this curved wave path distance can be used for any geodetic computations, it should be reduced to the surface of the spheroid by the application of both physical and geometric corrections. Figure 2.3 illustrates the situation.

2.3.2 The physical corrections, which involve the application of certain velocity corrections to the measured wave path distance, will not be further discussed in this manual. The geometric corrections reduce the wave path distance (d_1) firstly to the wave path chord (d_2), thence to the spheroidal chord distance (d_3), and finally to the spheroidal distance (s). All geometric corrections can be combined in the one rigorous formula to give the length of the spheroidal normal section (s) as follows:

$$s = 2R_\alpha \arc \sin \frac{d_3}{2R_\alpha}$$

$$\text{where } d_3 = \left[\frac{d_2^2 - (h_A - h_B)^2}{(1 + h_A/R_\alpha)(1 + h_B/R_\alpha)}\right]^{1/2}$$

2.3.3. The difference between the wave path length (d_1) and the wave path chord (d_2) is a function of the EDM measuring medium used and also of the meteorological conditions prevailing along the wave path at the time of measurement. This difference can usually be ignored for distance measurements of up to 15 kilometres in length using either light waves or microwaves.

2.3.4 The formulae given in paragraph *2.3.2*, which enable the wave path chord (d_2) to be directly reduced to the spheroidal distance (s), include the slope, spheroid level and chord-to-arc corrections. The slope correction reduces the wave path chord to a horizontal distance at the mean elevation of the terminals of the line. The spheroid level correction reduces the horizontal distance to the spheroidal chord distance (d_3) and the chord-to-arc correction when applied to the spheroidal chord distance gives the spheroidal distance (s).

$$\text{Slope correction} = (d_2^2 - \Delta h^2)^{1/2} - d_2$$

$$\text{Spheroid level correction} = \frac{h_m}{R}(d_2^2 - \Delta h^2)^{1/2}$$

$$\text{Chord-to-arc correction} = + \frac{d_3^3}{24R^2}$$

2.3.5 The chord-to-arc correction can usually be ignored for all but the most precise geodetic surveys. For a wave path distance (d_1) of 30 kilometres, the correction is 0·028 metre anywhere on the Australian National Spheroid. For a distance of 50 kilometres, the correction reaches approximately 0·13 metre.

The spheroid level correction can be re-written to give a "locality" or "line" height factor as follows:

$$\text{Height Factor} = 1 - \frac{h_m}{R + h_m}$$

If an accuracy of $1{:}10^5$ is required for the reduced distance, the mean height should be correct to 60 metres.

The value to be used for R in this formula can be computed from the formulae given in paragraph *3.2.3* or, again depending on the required accuracy, interpolated from the table below. For most practical purposes, the above formula can be used anywhere in Australia in the following form:

$$\text{Height Factor} = 1 - [h_m . 0{\cdot}1571 . 10^{-6}]$$

For computations anywhere on the NSW Integrated Survey Grid a value of R_m = 6 370 100 metres is in use. Values of 6 372 225 and 6 362 200 metres have been suggested for use anywhere within Victoria and Queensland respectively.

ϕ	R
0°	6 356 700
5	6 357 100
10	6 358 050
15	6 359 600
20	6 361 750
25	6 364 400
30	6 367 450
35	6 370 800
40	6 374 400
45	6 378 100
50	6 381 800
55	6 385 450

2.3.6 If geoidal, or orthometric, heights are used in any of the above formulae then h_m, h_A and h_B must become H_m, H_A and H_B respectively and the resulting distance is a sea level or geoidal distance, s′. The radius of curvature used in this computation is usually assumed to be equivalent to the radius of curvature of the spheroidal surface.

2.3.7 Further information on the reduction of measured distances to the surface of the spheroid, including the application of a combined point scale and height correction, is given in Chapter 5.

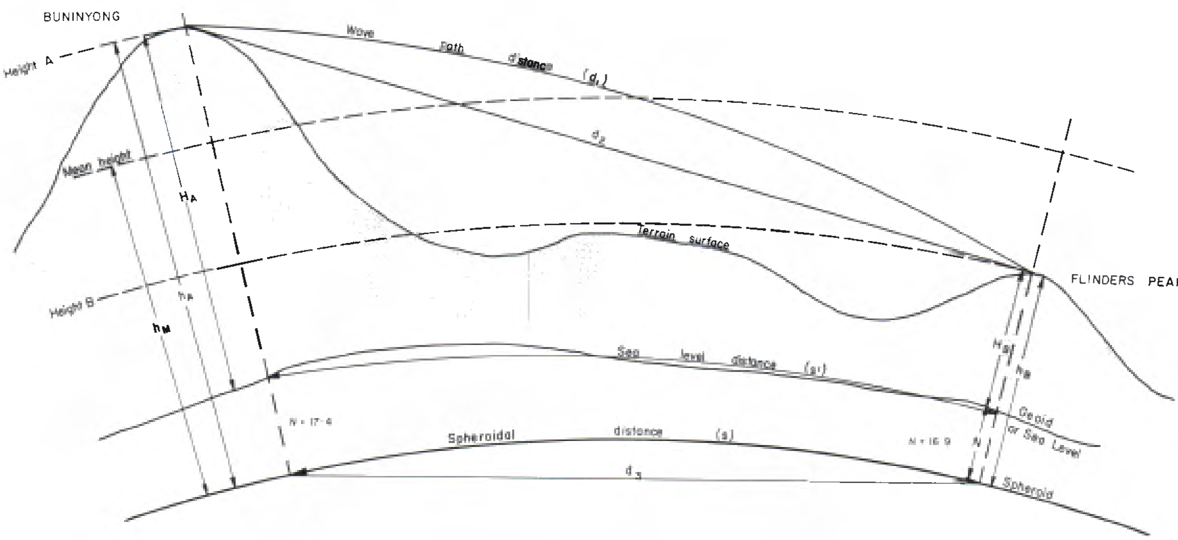


Figure 2.3 DIAGRAMMATIC REPRESENTATION OF THE REDUCTION OF A MEASURED DISTANCE TO THE SPHEROID

N values are shown in terms of N = + 4·9 metres at Johnston Geodetic Station

3.

RIGOROUS FORMULAE ON THE SPHEROID

3.1 AIMS

3.1.1 This chapter gives formulae and numerical examples for highly accurate computations on the spheroid:

The Direct Problem: latitude and longitude from spheroidal distance and azimuth

The Reverse Problem: spheroidal distance and azimuth from latitude and longitude

The formulae are those given by Dr A.R. Robbins in *Empire Survey Review* No. 125, 1962. They are accurate to better than 20 mm over distances of 1 500 kilometres. The errors can reach 16 metres at 4 500 kilometres, and more than 2 000 metres at 9 000 kilometres.

For further information on the highly accurate computation of the Direct and Reverse Problems on the spheroid, see "Long Geodesics on the Ellipsoid" by H.F. Rainsford published in the *Bulletin Geodesique,* No. 37, 1955.

3.1.2 It is not foreseen that these formulae will ever be used for manual computation: they require ten-figure trigonometrical functions, and it is easier and better to use the computer or calculator programs that are available. In the numerical examples, all trigonometrical functions and intermediate results are given, and these should be adequate for checking similar programs. They have not, however, been laid out in a form suitable for manual computation, and they may give a false impression of ease and brevity. A summary of rigorously computed numerical values for both of the test lines, using Robbins's formulae, is given in Annex H.

3.1.3 Robbins's formulae are for the normal section. For conversion to the geodesic, see paragraph *3.9*.

3.2 CONSTANTS FOR THE AUSTRALIAN NATIONAL SPHEROID AND THE WORLD GEODETIC SYSTEM 1972 SPHEROID

3.2.1 By definition (see paragraphs *1.2.1* and *1.6.2*).

	Australian National Spheroid	WGS 72 Spheroid
Major semi-axis, a	6 378 160 metres	6 378 135 metres
Flattening, f	1/298·25 exactly	1/298·26

3.2.2 From these figures can be derived:

	Australian National Spheroid	WGS 72 Spheroid
Flattening, f	0·003 352 891 869	0·003 352 779 454
Minor semi-axis, b = a(1 - f)	6 356 774·719 metres	6 356 750·520 metres
$e^2 = 2f - f^2 = (a^2 - b^2)/a^2$	0·006 694 541 855	0·006 694 317 778
e	0·081 820 179 996	0·081 818 810 663
$e'^2 = e^2/(1 - e^2) = (a^2 - b^2)/b^2$	0·006 739 660 796	0·006 739 433 689
e′	0·082 095 437 120	0·082 094 053 920

3.2.3 For the computation of radii of curvature on either of the above spheroids:

$$\rho = a(1 - e^2) / (1 - e^2 \sin^2 \phi)^{3/2}$$

$$\nu = a/(1 - e^2 \sin^2\phi)^{1/2}$$

$$R = (\rho\nu)^{1/2}$$

$$R\alpha = \rho\nu/(\rho \sin^2\alpha + \nu \cos^2\alpha)$$

3.3 THE DIRECT PROBLEM: ROBBINS'S FORMULAE

Robbins's formulae work in the normal section. In these paragraphs,
α_{12} is the azimuth at point 1 to point 2 in the normal section at point 1
α_{21} is the azimuth at point 2 to point 1 in the normal section at point 2
α'_{12} is the azimuth at point 1 to point 2 in the normal section at point 2
α'_{21} is the azimuth at point 2 to point 1 in the normal section at point 1

1. $h' = e' \cos \phi_1 \cos \alpha_{12}$
2. $g = e' \sin \phi_1$
3. $\eta = s/\nu_1$
4. $\sigma' = \eta \left\{[1 + \eta^2 h'^2(1 - h'^2)/6 - \eta^3 gh'(1 - 2h'^2)/8 - (\eta^4/120)[h'^2(4 - 7h'^2) - 3g^2(1 - 7h'^2)] + \eta^5 gh'/48]\right\}$

If any of ζ_2, $\Delta\lambda$ or α'_{21} approach 90° or 270°

then

5a. $\text{Cot } \Delta\lambda = (\cos \phi_1 \cot \sigma' - \sin \phi_1 \cos \alpha_{12})/\sin \alpha_{12}$
6a. $\text{Tan } \zeta_2 = (\sin \phi_1 \cos \Delta\lambda + \sin \Delta\lambda \cot \alpha_{12})/\cos \phi_1$
7a. $\text{Cot } \alpha'_{21} = (\cos \sigma' \cos \alpha_{12} - \sin \sigma' \tan \phi_1)/\sin \alpha_{12}$

else

5b. $\text{Sin } \zeta_2 = \sin \phi_1 \cos \sigma' + \cos \phi_1 \cos \alpha_{12} \sin \sigma'$
6b. $\text{Sin } \Delta\lambda = \sin \sigma' \sin \alpha_{12}/\cos \zeta_2$
7b. $\text{Sin } \alpha'_{21} = - \cos \phi_1 \sin \alpha_{12}/\cos \zeta_2$
8. $\mu = 1 + \tfrac{1}{2}e'^2 (\sin \zeta_2 - \sin \phi_1)^2$
9. $\text{Tan } \phi_2 = \tan \zeta_2(1 + e'^2)(1 - e^2\mu \sin \phi_1/\sin \zeta_2)$
10. $\lambda_2 = \lambda_1 + \Delta\lambda$
11. $\alpha_{21} = \alpha'_{21} - (\phi_2 - \zeta_2) \sin \alpha'_{21} \tan (\sigma'/2)$

3.4 THE REVERSE PROBLEM: ROBBINS'S FORMULAE

1. $\text{Tan } \zeta_2 = (1 - e^2)\tan \phi_2 + e^2\nu_1 \sin \phi_1/(\nu_2 \cos \phi_2)$
2. $\tau_1 = \cos \phi_1 \tan \zeta_2 - \sin \phi_1 \cos(\lambda_2 - \lambda_1)$
3. $\text{Tan } \alpha_{12} = \sin (\lambda_2 - \lambda_1)/\tau_1$
4. Obtain α_{21} from formulae 1-3 with suffixes 1 and 2 interchanged.

If Sin α_{12} is close to zero

then

5. $\chi = \tau_1/\cos \alpha_{12}$

else

5. $\chi = \sin (\lambda_2 - \lambda_1)/\sin \alpha_{12}$

6. $\text{Sin } \sigma' = \chi \cos \zeta_2$
7. $g = e' \sin \phi_1$
8. $h' = e' \cos \phi_1 \cos \alpha_{12}$
9. $s = \nu_1 \sigma' \left\{[1 - \sigma'^2 h'^2(1 - h'^2)/6 + \sigma'^3 gh'(1 - 2h'^2)/8 + \sigma'^4[h'^2(4 - 7h'^2) - 3g^2(1 - 7h'^2)]/120 - \sigma'^5 gh'/48]\right\}$

3.5 ROBBINS'S DIRECT FORMULAE: NUMERICAL EXAMPLE, AUSTRALIAN NATIONAL SPHEROID

From: Buninyong ϕ_1	= - 37° 39′ 15″.557 1	= - 0.657 191 886 3
λ_1	= +143° 55′ 30″.633 0	= + 2.511 968 194 8
To: Flinders Peak s	= 54 972.161 metres	
α_{12}	= 127° 10′ 27″.08	= + 2.219 608 319 7
Sin ϕ_1	= - 0.610 896 049 5	
Cos ϕ_1	= +0.791 710 816 3	
Sin α_{12}	= + 0.796 802 202 4	
Cos α_{12}	= - 0.604 240 225 7	
ν_1	= 6 386 142.439 metres	
h′	= - 0.039 273	
g	= - 0.050 152	
η	= 0.008 608 038 7	
η^2 term	= + 1·902 . 10^{-8}	
η^3 term	= + 1·566 . 10^{-10}	
η^4 term	= - 6·000 . 10^{-14}	
η^5 term	= + 1·940 . 10^{-15}	
Sum	= + 1·886 . 10^{-8}	
σ'	= 00° 29′ 35″·535 5	= 0·008 608 038 8
Sin σ'	= 0·008 607 932 5	
Cos σ'	= 0·999 962 951 1	
Sin ζ_2	= - 0·614 991 309 6	
ζ_2	= -37° 57′ 04″·639 8	= - 0·662 374 945 4
Cos ζ_2	= +0·788 533 885 9	
Tan ζ_2	= - 0·779 917 414 6	
Sin $\Delta\lambda$	= +0·008 698 192 6	
$\Delta\lambda$	= +00° 29′ 54″·153 6	= 0·008 698 302 3
λ_1	= 143° 55′ 30″·633 0	
λ_2	= **144° 25′ 24″·786 6**	= 2·520 666 497 1
Sin α'_{21}	= - 0·800 012 445 1	
α'_{21}	= 306° 52′ 07″·35	= 5·355 869 347 1
μ	= 1·000 000 056 5	
Tan ϕ_2	= - 0·779 952 416 8	
ϕ_2	= **- 37° 57′ 09″·128 8**	= - 0·662 396 709 0
ϕ_2 - ζ_2	= - 04″·489 0	= - 0·000 021 763 5
$\sigma'/2$	= +00° 14′ 47″·767 7	= 0·004 304 019 4
Tan $(\sigma'/2)$	= 0·004 304	
α_{21}	= **306° 52′ 07″·34**	= 5·355 869 272 1

3.6 ROBBINS'S REVERSE FORMULAE: NUMERICAL EXAMPLE, AUSTRALIAN NATIONAL SPHEROID

Station: 1. Buninyong

ϕ	= - 37°39'15"·557 1	= - 0·657 191 886 3
λ	= +143°55'30"·633 0	= + 2·511 968 194 8
$\Delta\lambda$	= + 0°29'54"·153 6	= + 0·008 698 302 3
Sin ϕ	= - 0·610 896 049 5	
Cos ϕ	= +0·791 710 816 3	
Tan ϕ	= - 0·771 615 136 4	
ν	= 6 386 142·439 metres	
Sin $\Delta\lambda$	= +0·008 698 192 4	
Cos $\Delta\lambda$	= +0·999 962 170 0	
Tan ζ	= - 0·771 649 998 2	
τ	= - 0·006 596 113 4	
Tan α	= - 1·318 684 487 5	
α	= **127°10'27"·08**	= 2·219 608 313 6

Station: 2. Flinders Peak

ϕ	= -37°57'09"·128 8	= - 0·662 396 708 8
λ	= +144°25'24"·786 6	= + 2·520 666 496 9
$\Delta\lambda$	= - 0°29'54"·153 6	= - 0·008 698 302 3
Sin ϕ	= - 0·615 008 470 6	
Cos ϕ	= +0·788 520 501 4	
Tan ϕ	= - 0·779 952 416 5	
ν	= 6 386 250·478 metres	
Sin $\Delta\lambda$	= - 0·008 698 192 4	
Cos $\Delta\lambda$	= +0·999 962 170 0	
Tan ζ	= - 0·779 917 414 3	
τ	= +0·006 523 361 3	
Tan α	= - 1·333 391 176 5	
α	= **306°52'07"·34**	= 5·355 869 266 2

Sin α_{12}	= +0·796 802 206 0	
Cos α_{12}	= - 0·604 240 220 9	
χ	= +0·010 916 375 9	
Cos ζ_2	= +0·788 533 886 0	
Sin σ'	= +0·008 607 932 3	
σ'	= +0°29'35"·535 4	= + 0·008 608 038 6
g	= - 0·050 152	
h'	= - 0·039 273	
σ'^2 term	= +1·902 . 10^{-8}	
σ'^3 term	= +1·566 . 10^{-10}	
σ'^4 term	= - 6·000 . 10^{-14}	
σ'^5 term	= +1·940 . 10^{-15}	
Sum	= - 1·886 . 10^{-8}	
s	= **54 972·161 metres**	

3.7 ROBBINS'S DIRECT FORMULAE: NUMERICAL EXAMPLE, WORLD GEODETIC SYSTEM 1972 SPHEROID

From: "M"	ϕ_1 = - 29° 03′ 23·″153 0	=	- 0·507 130 396 6
	λ_1 = +167° 57′ 06·″632 0	=	+2·931 312 631 6
To: "X"	s = 56 959·832 metres		
	α_{12} = 69° 37′ 50·″00	=	+1·215 282 454 4

Sin ϕ_1	= - 0·485 670 809 8		
Cos ϕ_1	= 0·874 141 787 4		
Sin α_{12}	= 0·937 467 747 9		
Cos α_{12}	= 0·348 072 149 9		
ν_1	= 6 383 176·604 metres		
h′	= 0·024 978		
g	= - 0·039 871		
η	= 0·008 923 430 4		
η^2 term	= $8{\cdot}275 \,.\, 10^{-9}$		
η^3 term	= $-8{\cdot}834 \,.\, 10^{-11}$		
η^4 term	= $-1{\cdot}192 \,.\, 10^{-13}$		
η^5 term	= $-1{\cdot}174 \,.\, 10^{-15}$		
Sum	= $8{\cdot}363 \,.\, 10^{-9}$		
σ'	= 0° 30′ 40·″589 7	=	0·008 923 430 5
Sin σ'	= 0·008 923 312 0		
Cos σ'	= 0·999 960 186 5		
Sin ζ_2	= - 0·482 936 427 2		
ζ_2	= -28° 52′ 38·″499 5	=	- 0·504 005 028 3
Cos ζ_2	= +0·875 655 415 8		
Tan ζ_2	= - 0·551 514 235 5		
Sin $\Delta\lambda$	= +0·009 553 206 7		
$\Delta\lambda$	= 0° 32′ 50·″520 3	=	0·009 553 352 1
λ_1	= 167° 57′ 06·″632 0		
λ_2	= **168° 29′ 57·″152 3**	=	2·940 865 983 6
Sin α'_{21}	= - 0·935 847 272 8		
α'_{21}	= 249° 21′ 55·″66	=	4·352 248 346 8
μ	= 1·000 000 025 2		
tan ϕ_2	= - 0·551 493 190 3		
ϕ_2	= **- 28° 52′ 35·″171 0**	=	- 0·503 988 891 4
$\phi_2 - \zeta_2$	= 0° 00′ 03·″328 5	=	0·000 016 137 0
$\sigma'/2$	= 0° 15′ 20·″294 8	=	0·004 461 715 2
Tan ($\sigma'/2$)	= 0·004 462		
α_{21}	= **249° 21′ 55·″68**	=	4·352 248 414 2

3.8 ROBBINS'S REVERSE FORMULAE: NUMERICAL EXAMPLE, WORLD GEODETIC SYSTEM 1972 SPHEROID

Station 1. "M"			*Station 2. "X"*		
ϕ	= - 29° 03′ 23·″153 0	= - 0·507 130 396 6	ϕ	= - 28° 52′ 35·″171 0	= - 0·503 988 891 2
λ	= +167° 57′ 06·″632 0	= +2·931 312 631 6	λ	= +168° 29′ 57·″152 3	= +2·940 865 983 6
$\Delta\lambda$	= +0° 32′ 50·″520 3	= +0·009 553 352 0	$\Delta\lambda$	= - 0° 32′ 50·″520 3	= - 0·009 553 352 0
Sin ϕ	= - 0·485 670 809 8		Sin ϕ	= - 0·482 922 296 6	
Cos ϕ	= +0·874 141 787 4		Cos ϕ	= +0·875 663 208 9	
Tan ϕ	= 0·555 597 291 8		Tan ϕ	= - 0·551 493 190 1	
ν	= 6 383 176·604 metres		ν	= 6 383 119·636 metres	
Sin $\Delta\lambda$	= +0·009 553 206 7		Sin $\Delta\lambda$	= - 0·009 553 206 7	
Cos $\Delta\lambda$	= +0·999 954 367 1		Cos $\Delta\lambda$	= +0·999 954 367 1	
Tan ζ	= - 0·555 576 210 2		Tan ζ	= - 0·551 514 235 3	
τ	= +0·003 547 007 8		τ	= - 0·003 597 387 6	
Tan α	= +2·693 314 288 4		Tan α	= +2·655 595 575 9	
α	= **69° 37′ 50·″00**	= +1·215 282 435 9	α	= **249° 21′ 55·″68**	= +4·352 248 395 7

Sin α_{12}	= +0·937 467 741 5	
Cos α_{12}	= +0·348 072 167 3	
χ	= +0·010 190 437 8	
Cos ζ_2	= +0·875 655 415 9	
Sin σ'	= +0·008 923 312 0	
σ'	= +0° 30′ 40·″5897	= +0·008 923 430 5
g	= - 0·039 871	
h′	= +0·024 978	
σ'^2 term	= $+8{\cdot}275 \, . \, 10^{-9}$	
σ'^3 term	= $-8{\cdot}834 \, . \, 10^{-11}$	
σ'^4 term	= $-1{\cdot}192 \, . \, 10^{-13}$	
σ'^5 term	= $-1{\cdot}174 \, . \, 10^{-15}$	
Sum	= $-8{\cdot}363 \, . \, 10^{-9}$	
s	= **56 959·832 metres**	

3.9 CONVERSION FROM THE NORMAL SECTION TO THE GEODESIC

3.9.1 A plane containing the spheroidal normal at a point A and the point B intersects the surface of the spheroid along the normal section A-B. The reciprocal normal sections from A to B and from B to A do not coincide although the distance along each is the same.

3.9.2 A geodesic is a unique surface curve and is the line of shortest distance between two points on the spheroid. A geodesic generally lies between the two reciprocal normal sections. The mid-point of a geodesic 150 kilometres in length lies mid-way between the two normal sections and less than 2·5 centimetres from either.

3.9.3 Angles and azimuths are observed in normal sections. It is seldom necessary to compute geodesic azimuths, but if required they can be computed from the normal section by adding:

$$-[(e'^2/12)(s/R)^2 (\cos^2\phi \sin 2\,\alpha_{12}) + (e'^2/48)(s/R)^3 (\sin\alpha_{21} \sin 2\phi)]''$$

Any reasonable value can be used for R. At 1 500 kilometres, this correction can attain 7″, and the formula is accurate to about 0·6″. At greater distances the accuracy of the formula deteriorates, the correction reaching 1′ 31″ with errors up to 2·″5 at 4 500 kilometres, and 4′ 40″ with errors up to 44″ at 9 000 kilometres. The above formula can be simplified as follows, using s in kilometres:

$$-[0{\cdot}028(s/100)^2 \sin 2\,\alpha \cos^2\phi_1]''$$

3.9.4 The difference between the length of the geodesic and either of the normal sections seldom attains 1 mm at 1 500 kilometres and can usually be ignored. If required, this difference can be computed from the following formula:

$$s = s_G\left(1 + \frac{1}{360} e^4 \frac{s^4}{a^4} \cos^4\phi_1 \sin^2 2\,\alpha_{12} + ...\right)$$

where s_G = length of geodesic.

3.9.5 The following table shows the corrections required to convert a normal section azimuth and distance to an equivalent geodesic azimuth and distance.

CORRECTIONS FROM NORMAL SECTION TO GEODESIC (AZIMUTH 45°)

At	*Distance*	*Azimuth*	*Azimuth (Simplified Formula)*
1 000 km	- 0·000 04 metre	+2·″33	+ 2·″1
1 500 km	- 0·000 32 metre	+5·″46	+ 4·″7
2 500 km	- 0·004 13 metre	+16·″21	+13·″1

For the test line from Buninyong to Flinders Peak the corrections required to convert the normal section azimuth and distance to the geodesic are - 1·46 . 10^{-11} metres and + 0·″005 146 respectively. Using the simplified formula, the azimuth correction is +0·″005 098.

4.

RIGOROUS FORMULAE BETWEEN SPHEROID AND GRID

4.1 AIMS

4.1.1 The aims of this chapter are to provide:

- the adopted formulae for meridian distance;
- Redfearn's formulae for obtaining easting, northing, grid convergence and point scale factor, from latitude and longitude;
- Redfearn's formulae for the reverse computation, from grid to spheroid;
- numerical examples.

4.1.2 Redfearn's formulae were published in *Empire Survey Review* No. 69, 1948. They are accurate to better than 1 mm in any zone of the Australian Map Grid. For purposes of definition, they are to be regarded as exact, and not merely as the opening terms of an infinite series. All angles are in radians. For the definition of symbols, see paragraph *2.1*.

4.1.3 In the numerical examples, all trigonometrical functions and intermediate results are given. They should be adequate for checking other computer programs. They have not been laid out in a form specifically designed for manual computation, and they may give a false idea of ease and brevity. A summary of rigorously computed numerical values for both of the test lines, using Redfearn's formulae, is given in Annex H. The formula for line scale factor is given in paragraph *5.3.1*.

4.2 MERIDIAN DISTANCE—RIGOROUS METHOD

The length of an arc of a meridian is given by

$$a(1 - e^2)\int_{\phi_1}^{\phi_2}[1 - (e^2\sin^2\phi)]^{-3/2}d\phi$$

where ϕ_1 and ϕ_2 are the starting and finishing points.

When calculating the meridian distance from the equator, ϕ_1 becomes zero.

This formula cannot be solved directly but may be evaluated by a method such as Simpson's Rule; however due to the iterations required it is relatively inefficient.

4.3 MERIDIAN DISTANCE—SERIES METHOD

A more efficient method is to use the series expansion of the formula in paragraph *4.2* which reduces to

$$m = a(A_0\phi - A_2\sin 2\phi + A_4\sin 4\phi - A_6\sin 6\phi)$$

where
$$A_0 = 1 - e^2/4 - 3e^4/64 - 5e^6/256$$
$$A_2 = (3/8)(e^2 + e^4/4 + 15e^6/128)$$
$$A_4 = (15/256)(e^4 + 3e^6/4)$$
$$A_6 = 35e^6/3072$$

This limited formula is correct to less than 0·5 mm in latitude 45°.

On the Australian National Spheroid this formula becomes

$$m = 111\ 133{\cdot}348\ 785\ (57{\cdot}295\ 779\ 51\ \phi) - 16\ 038{\cdot}954\ 6\ \sin 2\phi + 16{\cdot}833\ 1\ \sin 4\phi - 0{\cdot}021\ 8\ \sin 6\phi$$

and on the WGS 72 Spheroid

$$m = 111\ 132{\cdot}919\ 438\ (57{\cdot}295\ 779\ 51\ \phi) - 16\ 038{\cdot}354\ 0\ \sin 2\phi + 16{\cdot}831\ 9\ \sin 4\phi - 0{\cdot}021\ 8\ \sin 6\phi$$

where ϕ is in radians.

4.4 FOOT-POINT LATITUDE

The foot-point latitude is the latitude for which the meridian distance is equal to N'/k_0. This value can be calculated directly provided three other values namely n, G and σ are calculated first.

$$n = \frac{a - b}{a + b} = f/(2 - f)$$

On the Australian National Spheroid

$$n = 0{\cdot}001\ 679\ 261\ 1$$

and on the WGS 72 Spheroid

$$n = 0{\cdot}001\ 679\ 204\ 7$$

G = mean length of an arc of one degree of the meridian

$$= a(1 - n)(1 - n^2)(1 + \frac{9}{4}n^2 + \frac{225}{64}n^4)\frac{\pi}{180}$$

On the Australian National Spheroid

$$G = 111\ 133{\cdot}348\ 785$$

and on the WGS 72 Spheroid

$$G = 111\ 132{\cdot}919\ 438$$

$$\sigma = \frac{m.\pi}{G.180}$$

The formula for foot-point latitude is then, in radians:

$$\phi' = \sigma + \left(\frac{3n}{2} - \frac{27n^3}{32}\right) \sin 2\sigma + \left(\frac{21n^2}{16} - \frac{55n^4}{32}\right) \sin 4\sigma + \left(\frac{151n^3}{96}\right) \sin 6\sigma + \left(\frac{1097n^4}{512}\right) \sin 8\sigma$$

On the Australian National Spheroid this becomes, in radians:

$$\phi' = \sigma + 0{\cdot}002\,518\,887\,7 \sin 2\sigma + 0{\cdot}000\,003\,701\,1 \sin 4\sigma + 0{\cdot}000\,000\,007\,4 \sin 6\sigma + (1{\cdot}7 \,.\, 10^{-11}) \sin 8\sigma$$

and on the WGS 72 Spheroid, in radians:

$$\phi' = \sigma + 0{\cdot}002\,518\,803\,1 \sin 2\sigma + 0{\cdot}000\,003\,700\,9 \sin 4\sigma + 0{\cdot}000\,000\,007\,4 \sin 6\sigma + (1{\cdot}7 \,.\, 10^{-11}) \sin 8\sigma$$

4.5 SPHEROID TO GRID: REDFEARN'S FORMULAE

4.5.1 EASTING

$$E' = k_0 \left\{ \nu \omega \cos \phi + \nu \frac{\omega^3}{6} \cos^3 \phi (\psi - t^2) + \nu \frac{\omega^5}{120} \cos^5 \phi [4\psi^3(1 - 6t^2) + \psi^2(1 + 8t^2) - \psi(2t^2) + t^4] + \nu \frac{\omega^7}{5040} \cos^7 \phi (61 - 479t^2 + 179t^4 - t^6) \right\}$$

4.5.2 NORTHING

$$N' = k_0 \left\{ m + \nu \sin \phi \frac{\omega^2}{2} \cos \phi + \nu \sin \phi \frac{\omega^4}{24} \cos^3 \phi (4\psi^2 + \psi - t^2) + \nu \sin \phi \frac{\omega^6}{720} \cos^5 \phi [8\psi^4(11 - 24t^2) - 28\psi^3(1 - 6t^2) + \psi^2(1 - 32t^2) - \psi(2t^2) + t^4] + \nu \sin \phi \frac{\omega^8}{40320} \cos^7 \phi (1385 - 3111t^2 + 543t^4 - t^6) \right\}$$

4.5.3 GRID CONVERGENCE (radians)

$$\gamma = -\sin \phi\, \omega - \sin \phi \frac{\omega^3}{3} \cos^2 \phi (2\psi^2 - \psi) - \sin \phi \frac{\omega^5}{15} \cos^4 \phi [\psi^4(11 - 24t^2) - \psi^3(11 - 36t^2) + 2\psi^2(1 - 7t^2) + \psi t^2] - \sin \phi \frac{\omega^7}{315} \cos^6 \phi (17 - 26t^2 + 2t^4)$$

4.5.4 POINT SCALE FACTOR

$$k = k_o \left\{ 1 + \frac{\omega^2}{2} \cos^2 \phi\, \psi + \frac{\omega^4}{24} \cos^4 \phi [4\psi^3(1 - 6t^2) + \psi^2(1 + 24t^2) - 4\psi t^2] + \frac{\omega^6}{720} \cos^6 \phi (61 - 148t^2 + 16t^4) \right\}$$

4.6 FROM AUSTRALIAN GEODETIC DATUM TO AUSTRALIAN MAP GRID: NUMERICAL EXAMPLE

Station: *BUNINYONG*

Latitude ϕ = -37°39′15·″557 1 Longitude λ = +143°55′30·″633 0

Zone: 54 λ_0 = 141° ω = + 2°55′30·″633 0

Functions:

ϕ radians = - 0·657 191 886 3			
sin ϕ = - 0·610 896 049 5			A_0 =0·998 324 257 9
sin 2ϕ = - 0·967 306 020 1		e^2 = 0·006 694 541 9	A_2 = 0·002 514 668 0
sin 4ϕ = - 0·490 640 893 2		e^4 = 0·000 044 816 9	A_4 = 0·000 002 639 2
sin 6ϕ = +0·718 441 151 0		e^6 = 0·000 000 300 0	A_6 = 0·000 000 003 4

Meridian Distance:

1st term = - 4 184 650·835
2nd term = + 15 514·577
3rd term = - 8·259
4th term = - 0·016
Sum = m = - 4 169 144·533

Radii of Curvature:

ρ = 6 359 277·924
ν = 6 386 142·439

Powers:

Power	cos ϕ	ω	t = tan ϕ	$\psi = \nu / \rho$
1	0·791 710 816 3	0·051 053 949 5	– 0·771 615 136 4	1·004 224 459 9
2	0·626 806 016 7	0·002 606 505 8	0·595 389 918 8	1·008 466 765 9
3	0·496 249 103 1	0·000 133 072 4		1·012 726 993 4
4	0·392 885 782 5	0·000 006 793 9	0·354 489 155 4	1·017 005 218 0
5	0·311 051 923 6	0·000 000 346 9		
6	0·246 263 172 3	0·000 000 017 7	0·211 059 269 4	
7	0·194 969 217 2	0·000 000 000 9		
8		(+4·6 . 10^{-11})		

Easting:

1st term	=	+ 258 127·648
2nd term	=	+ 28·736
3rd term	=	- 0·031
4th term	=	(-3·6 . 10^{-5})
Sum	=	+ 258 156·352
Sum . k_0 = E′	=	+ 258 053·090
False Origin	=	+ 500 000·000
E	=	**758 053·090**

Northing:

m	=	- 4 169 144·533
1st term	=	- 4 025·327
2nd term	=	- 2·435
3rd term	=	- 0·001
4th term	=	(+2·4 . 10^{-7})
Sum	=	- 4 173 172·295
Sum . k_0 = N′	=	- 4 171 503·027
False Origin	=	+ 10 000 000·000
N	=	**5 828 496·973**

Grid Convergence:

1st term	= + 1°47′13·″12	=	+0·031 188 656 1
2nd term	= + 3·″55	=	+0·000 017 200 9
3rd term	= (+2·0 . 10^{-3})	=	(+0·793 8 . 10^{-8})
4th term	= (+2·0 . 10^{-7})	=	(+0·962 4 . 10^{-12})
Sum = γ	= **+ 1°47′16·″67**	=	+0·031 205 864 9

Point Scale:

1st term	=	+ 1·000 820 34
2nd term	=	+ 0·000 000 29
3rd term	=	(-1·3 . 10^{-10})
Sum	=	+ 1·000 820 63
Sum . k_0 = k	=	**1·000 420 30**

4.7 FROM WORLD GEODETIC SYSTEM 1972 DATUM TO UNIVERSAL TRANSVERSE MERCATOR GRID: NUMERICAL EXAMPLE

Station: "M"

Latitude ϕ = -29°03′23·″153 0 Longitude λ = +167°57′06·″632 0

Zone: 58 λ_0 = 165° ω = + 2°57′06·″632 0

Functions:

ϕ radians = - 0·507 130 396 6			
sin ϕ = - 0·485 670 809 8			A_o = 0·998 324 314 0
sin 2ϕ = - 0·849 090 299 5		e^2 = 0·006 694 317 8	A_2 = 0·002 514 583 7
sin 4ϕ = - 0·897 060 045 0		e^4 = 0·000 044 813 9	A_4 = 0·000 002 639 0
sin 6ϕ = - 0·098 649 563 8		e^6 = 0·000 000 300 0	A_6 = 0·000 000 003 4

Meridian Distance:

1st term = -3 229 126·045
2nd term = + 13 618·007
3rd term = - 15·099
4th term = + 0·002
Sum = m = -3 215 523·135

Radii of Curvature:

ρ = 6 350 473·178
ν = 6 383 176·604

Powers:

Power	cos ϕ	ω	t = tan ϕ	$\psi = \nu / \rho$
1	0·874 141 787 4	0·051 519 365 8	- 0·555 597 291 8	1·005 149 762 1
2	0·764 123 864 6	0·002 654 245 1	0·308 688 350 7	1·010 326 044 3
3	0·667 952 600 8	0·000 136 745 0		1·015 528 983 1
4	0·583 885 280 4	0·000 007 045 0	0·095 288 497 8	1·020 758 715 8
5	0·510 398 522 7	0·000 000 363 0		
6	0·446 160 676 9	0·000 000 018 7	0·029 414 449 2	
7	0·390 007 691 6	0·000 000 001 0		
8		(4·96 . 10^{-11})		

Easting:

1st term	=	+287 467·830
2nd term	=	+ 67·677
3rd term	=	- 0·005
4th term	=	(-3·32 . 10^{-5})
Sum	=	+287 535·502
Sum . k_o = E′	=	+287 420·487
False Origin	=+	500 000·000
E	=	**787 420·487**

Northing:

m	=	- 3 215 523·135
1st term	=	- 3 596·431
2nd term	=	- 2·880
3rd term	=	- 0·002
4th term	=	(-7·09 . 10^{-7})
Sum	=	- 3 219 122·448
Sum . k_o= N′	=	- 3 217 834·799
False Origin	=	+10 000 000·000
N	=	**6 782 165·201**

Grid Convergence:

1st term	=	+1°26′01·″05	=	0·025 021 452 1
2nd term	=	+0°00′03·″54	=	0·000 017 178 2
3rd term	=	(+2·47.10^{-3})	=	(1·197 5.10^{-8})
4th term	=	(+1·25.10^{-6})	=	(6·073.10^{-12})
Sum = γ	=	**+1°26′04·″59**	=	0·025 038 642 2

Point Scale:

1st term	=	+1·001 019 308 3
2nd term	=	+0·000 000 650 1
3rd term	=	(1·951.10^{-10})
Sum	=	+1·001 019 958 6
Sum . k_0 = k	=	**1·000 619 550 6**

4.8 GRID TO SPHEROID: REDFEARN'S FORMULAE

4.8.1 LATITUDE (radians)

Let $x = E'/k_o\nu'$

$$\phi = \phi' - (t'/k_0\rho')\, xE'/2$$
$$+ (t'/k_0\rho')\,(x^3E'/24)\,[-4\psi'^2 + 9\psi'(1 - t'^2) + 12t'^2]$$
$$- (t'/k_0\rho')\,(x^5E'/720)\,[8\psi'^4(11 - 24t'^2) - 12\psi'^3(21 - 71t'^2)$$
$$+ 15\psi'^2(15 - 98t'^2 + 15t'^4) + 180\psi'(5t'^2 - 3t'^4) + 360t'^4]$$
$$+ (t'/k_0\rho')\,(x^7E'/40320)\,(1385 + 3633t'^2 + 4095t'^4 + 1575t'^6)$$

4.8.2 LONGITUDE (radians)

Let $x = E'/k_o\,\nu'$

$$\omega = \sec\phi' x$$
$$- \sec\phi'\, x^3/6\, (\psi' + 2t'^2)$$
$$+ \sec\phi'\, x^5/120\, [-4\psi'^3(1 - 6t'^2) + \psi'^2(9 - 68t'^2) + 72\psi' t'^2 + 24t'^4]$$
$$- \sec\phi'\, x^7/5040\, (61 + 662t'^2 + 1320t'^4 + 720t'^6)$$

4.8.3 GRID CONVERGENCE (radians)

Let $x = E'/k_o\nu'$

$$\gamma = -t'x$$
$$+t'x^3/3\, (-2\psi'^2 + 3\psi' + t'^2)$$
$$-t'x^5/15\, [\psi'^4(11 - 24t'^2) - 3\psi'^3(8 - 23t'^2) + 5\psi'^2(3 - 14t'^2) + 30\psi' t'^2 + 3t'^4]$$
$$+t'x^7/315\, (17 + 77t'^2 + 105t'^4 + 45t'^6)$$

4.8.4 POINT SCALE FACTOR

Let $x = E'^2/k_o^2\rho'\nu'$

$$k = k_0 \left\{1 + x/2 + x^2/24\, [4\psi'(1 - 6t'^2) - 3(1 - 16t'^2) - 24t'^2/\psi'] + x^3/720\right\}$$

Erecting antenna for satellite survey equipment, Antarctica *Photo: R. Butler, Antarctic Division* *Department of Science*

4.9 FROM AUSTRALIAN MAP GRID TO AUSTRALIAN GEODETIC DATUM: NUMERICAL EXAMPLE

Station:	*BUNINYONG*	*Zone:* 54; k_0 =	0·999 6
E	= 758 053·090	N =	5 828 496·973
False Origin	= - 500 000·000	False Origin =	- 10 000 000·000
E′	= 258 053·090	N′ =	- 4 171 503·027
E'/k_0	= + 258 156·353	$N'/k_0 = m$ =	- 4 173 172·296

$$\sigma = \frac{m \cdot \pi}{G \cdot 180} = -0{\cdot}655\ 389\ 202\ 4$$

2σ	= -1·310 778 404 7
4σ	= -2·621 556 809 5
6σ	= -3·932 335 214 2
8σ	= -5·243 113 619 0

Foot-Point Latitude:

1st term	= - 0·655 389 202 4
2nd term	= - 0·002 434 216 2
3rd term	= - 0·000 001 839 1
4th term	= +0·000 000 005 3
5th term	= $(1{\cdot}47 \cdot 10^{-11})$
ϕ'	= - 0·657 825 252 4
	= -37° 41′ 26·″198 2
Sec ϕ'	= 1·263 705 299 9
$t'/k_0\rho'$	= - 0·000 000 121 5

Radii of Curvature:

ρ' = 6 359 317·152

ν' = 6 386 155·570

Powers:

Power	η	$E'/k_0\nu'$	$E'^2/k_0^2\rho'\nu'$	$t'=\tan\phi'$	$\psi'=\nu'/\rho'$
1	0·001 679 261 1	0·040 424 375 8	0·001 641 026 7	- 0·772 626 096 5	1·004 220 330 1
2	0·000 002 819 9		0·000 002 693 0	0·596 951 085 0	1·008 458 471 5
3	0·000 000 004 7	0·000 066 058 7	0·000 000 004 4		1·012 714 499 1
4	$(7{\cdot}95 \cdot 10^{-12})$			0·356 350 597 9	1·016 988 488 7
5		0·000 000 107 9			
6				0·212 723 876 0	
7		0·000 000 000 2			

Latitude:

ϕ'	= -37° 41′ 26·″198 2	= - 0·657 825 252 4
1st term	= + 2 10· 761 6	= + 0·000 633 950 0
2nd term	= - 0· 120 6	= - 0·000 000 584 7
3rd term	= + 0· 000 1	= $(6{\cdot}122 \cdot 10^{-10})$
4th term	= $(-1{\cdot}5 \cdot 10^{-7})$	= $(-7{\cdot}339 \cdot 10^{-13})$
Sum = ϕ	= **- 37° 39′ 15·″557 1**	= - 0·657 191 886 4

Longitude:

λ_0	= + 141° 00′ 00·″000 0	
1st term	= + 2 55 36· 934 1	= 0·051 084 497 9
2nd term	= - 6· 308 1	= - 0·000 030 582 7
3rd term	= + 0· 007 1	= 0·000 000 034 5
4th term	= $(-9{\cdot}9 \cdot 10^{-6})$	= $(-4{\cdot}776 \cdot 10^{-11})$
Sum = λ	= **+ 143° 55′ 30·″633 0**	

Grid Convergence:

1st term	= + 1° 47′ 22·″25	= 0·031 232 927 7
2nd term	= - 5· 59	= - 0·000 027 096 3
3rd term	= + 0· 01	= $(3{\cdot}372 \cdot 10^{-8})$
4th term	= $(-9{\cdot}8 \cdot 10^{-6})$	= $(-4{\cdot}757 \cdot 10^{-11})$
Sum = γ	= + **1° 47′16·″67**	= 0·031 205 865 0

Point Scale:

1st term	= + 1·000 820 51
2nd term	= + 0·000 000 11
3rd term	= $(+6{\cdot}1 \cdot 10^{-12})$
Sum	= + 1·000 820 63
Sum . k_0 = k	= **1·000 420 30**

4.10 FROM UNIVERSAL TRANSVERSE MERCATOR GRID TO WORLD GEODETIC SYSTEM 1972 DATUM: NUMERICAL EXAMPLE

Station:	"*M*"	*Zone:* 58; k_0 =	0·999 6
E	= 787 420·487	N	= 6 782 165·201
False Origin	= -500 000·000	False Origin	= -10 000 000·000
E′	= + 287 420·487	N′	= - 3 217 834·799
E'/k_o	= + 287 535·501	$N'/k_o = m$	= - 3 219 122·448

$$\sigma = \frac{m \cdot \pi}{G \cdot 180} = -0{\cdot}505\,559\,343\,1$$

2σ	= -1·011 118 686 1
4σ	= -2·022 237 372 2
6σ	= -3·033 356 058 4
8σ	= -4·044 474 744 5

Foot-Point Latitude:

1st term	= - 0·505 559 343 1
2nd term	= - 0·002 134 500 0
3rd term	= - 0·000 003 330 1
4th term	= - 0·000 000 000 8
5th term	= $(+1{\cdot}337 \,.\, 10^{-11})$
ϕ'	= - 0·507 697 174 0
	= - 29°05′20·″059 2
Sec ϕ'	= 1·144 339 710 4
$t'/k_o\rho'$	= - 0·000 000 087 6

Radii of Curvature

ρ'	= 6 350 503·925
ν'	= 6 383 186·906

Powers:

Power	η	$E'/k_0\nu'$	$E'^2/k_0^2\rho'\nu'$	$t'=\tan\phi'$	$\psi'=\nu'/\rho'$
1	0·001 679 204 7	0·045 045 759 3	0·002 039 563 3	- 0·556 339 260 4	1·005 146 517 7
2	0·000 002 819 7		0·000 004 159 8	0·309 513 372 7	1·010 319 521 9
3	0·000 000 004 7	0·000 091 403 3	0·000 000 008 5		1·015 519 149 2
4	$(7{\cdot}951 \,.\, 10^{-12})$			0·095 798 527 9	1·020 745 536 4
5		0·000 000 185 5			
6				0·029 650 925 5	
7		0·000 000 000 4			

Latitude:

ϕ'	= - 29°05′20·″059 2	= - 0·507 697 174 0
1st term	= + 01 57· 023 2	= + 0·000 567 344 6
2nd term	= - 0· 117 1	= - 0·000 000 567 9
3rd term	= + 0· 000 1	= $(+6{\cdot}032 \,.\, 10^{-10})$
4th term	= $(-1{\cdot}43 \,.\, 10^{-7})$	= $(-6{\cdot}932 \,.\, 10^{-13})$
Sum = ϕ	= **- 29°03′ 23·″153 0**	= - 0·507 130 396 6

Grid Convergence:

1st term	= + 1°26′ 09·″15	= + 0·025 060 724 4
2nd term	= - 04· 56	= - 0·000 022 108 7
3rd term	= + 00· 01	= $(+2{\cdot}646 \,.\, 10^{-8})$
4th term	= $(-7{\cdot}16 \,.\, 10^{-6})$	= $(-3{\cdot}471 \,.\, 10^{-11})$
Sum = γ	= + **1°26′ 04·″59**	= + 0·025 038 642 2

Longitude:

λ_0	= + 165°00′00·″000 0	
1st term	= + 2 57 12· 466 3	= +0·051 547 651 2
2nd term	= - 05· 840 1	= - 0·000 028 313 8
3rd term	= + 0· 005 8	= +0·000 000 028 3
4th term	= $(-7{\cdot}29 \,.\, 10^{-6})$	= $(-3{\cdot}535 \,.\, 10^{-11})$
Sum = λ	= **167°57′ 06·″632 0**	

Point Scale:

1st term	= + 1·001 019 78
2nd term	= + 0·000 000 18
3rd term	= $(+1{\cdot}18 \,.\, 10^{-11})$
Sum	= + 1·001 019 96
Sum . k_o = k	= **1·000 619 55**

5.

FORMULAE ON THE GRID

5.1 AIMS

5.1.1 Although the Australian Map Grid and the Universal Transverse Mercator Grid correspond, coordinates on the former, as delineated in Annex A, are derived from a Transverse Mercator projection of latitudes and longitudes on the Australian Geodetic Datum. Coordinates on the UTM Grid, which is used for all mapping of Australian offshore islands and external territories lying outside the limits of the AMG, are derived from a Transverse Mercator projection of latitudes and longitudes on the WGS 72 geodetic datum.

5.1.2 The aims of this chapter are, therefore, to provide:

- working formulae for computation on both the AMG and the UTM Grid without the need for reference to tables of latitude functions;
- a method for the computation of latitude functions;
- formulae for obtaining grid bearing and spheroidal distance from AMG (or UTM Grid) coordinates;
- formulae for obtaining AMG (or UTM Grid) coordinates from grid bearing and spheroidal distance;
- methods of traverse computation using arc-to-chord corrections and line scale factors;
- a description of the methods used for zone to zone transformations;
- numerical examples.

5.1.3 Terms in the given formulae can be used in their entirety for first order accuracy or truncated with negligible effect for most practical purposes.

5.2 LATITUDE FUNCTIONS

5.2.1 VALUES FOR $1/6r^2$ AND $1/6r^2 \sin 1''$

Arc-to-chord corrections, δ, and line scale factors, K, form an integral part of computations on the AMG or UTM Grid, and require a knowledge of the values of $1/6r^2$ and $1/6r^2 \sin 1''$.

Depending upon the accuracy required for δ and K, and also upon the location and orientation of the subject line in relation to the grid zone central meridian, r^2 can be computed from:

(1) the mean latitude, ϕ_m, for each traverse line as determined from the known coordinates of the terminals.

(2) the mean latitude of the area under survey, possibly scaled from a good quality map.

(3) the approximate value for latitude given by the first and second terms of the formulae for foot-point latitude in paragraph *4.4*.

Method (1) need only be used where the highest precision is required. Method (2) can be used for most purposes, possibly in conjunction with the formula for directly obtaining the function $1/6r^2$ as given in paragraph *5.2.2*. Method (3), which is used in this chapter, substitutes the approximate latitude value into the formula for σ and ν given in paragraph *3.2.3* from which r^2 ($= \rho\nu k_0^2$) is then computed. The approximate latitude value can be computed in terms of the northing, N, of the given or known station or the mean northing of the line.

The table below shows that the accuracy of ϕ_m only becomes critical on or near the grid zone boundaries. For the determination of δ, the accuracy required for ϕ_m is much less critical.

	E (km)					
	50	100	200	250	300	350
For K to 0·1 ppm	14°	3·5°	0° 52′	0° 33′	0° 23′	0° 17′
For K to 1 ppm		35°	8·7°	5·5°	3·8°	2·8°

5.2.2 COMPUTATION OF LATITUDE FUNCTIONS: NUMERICAL EXAMPLES

The following examples show the suggested method of computation of latitude functions.

AMG/ANS		UTM/WGS 72	
Station: BUNINYONG		*Station:* "M"	
N (Zone 55) =	5 828 074·208	N (Zone 58) =	6 782 165·201
False Origin =	-10 000 000·000	False Origin =	-10 000 000·000
N′ =	- 4 171 925·792	N′ =	- 3 217 834·799
$N'/k_0 = m$ =	- 4 173 595·230	$N'/k_0 = m$ =	- 3 219 122·448

Calculate a first approximation of the latitude for use in the formula for meridian distance, rounding off to the nearest unit.

ANS

$$\phi' \text{(approx)} = \frac{N'/k_0}{G} = \frac{-4\ 173\ 595}{111\ 133} = -37°·55$$

$$\phi' = \frac{N'/k_0 + 160\ 39 \sin 2\phi}{111\ 133} = -37°\ 42'$$

WGS 72

$$= \frac{-3\ 219\ 122}{111\ 133} = -28°·97$$

$$= \frac{N'/k_0 + 160\ 38 \sin 2\phi}{111\ 133} = -29°\ 05'$$

These values for latitude are within approximately 2 minutes of the rigorously computed values.

	ANS	WGS 72
$\rho = \frac{a(1 - e^2)}{(1 - e^2 \sin^2\phi)^{3/2}}$ =	6 359 330	6 350 500
$\nu = \frac{a}{(1 - e^2 \sin^2\phi)^{1/2}}$ =	6 386 160	6 383 190

By definition, $r^2 = \rho\nu k_0^2$. Alternatively, $r^2 = \frac{k_0^2 a^2(1 - e^2)}{(1 - e^2\sin^2\phi)^2}$

r^2 =	$4·057\ 92.10^{13}$	$4·050\ 40.10^{13}$
$1/6r^2$ =	$4·107\ 19.10^{-15}$	$4·114\ 82.10^{-15}$
$1/6r^2 \sin 1''$ =	$8·471\ 69.10^{-10}$	$8·487\ 42.10^{-10}$

Calculation of latitude functions can be simplified by use of the following approximate formula which will generally be sufficiently accurate for use anywhere within the area covered by the AMG or the UTM Grid between 8° and 45° South as, within this region, the values computed from this formula will never differ from the rigorously computed values by more than one unit in the fourth decimal place.

$$1/6\,r^2 = \left[4·072\ 27 + \frac{\sin^2(90 - \phi)}{18}\right].10^{-15}$$

where $\phi = N'.9.10^{-6}$

Latitude functions computed from the approximate formula for the above examples are as follows:

	BUNINYONG	*"M"*
$1/6r^2$ =	$4·107\ 19.10^{-15}$	$4·114\ 80.10^{-15}$
$1/6r^2 \sin 1''$ =	$8·471\ 69.10^{-10}$	$8·487\ 39.10^{-10}$

5.3 GRID BEARINGS AND SPHEROIDAL DISTANCE FROM AMG (OR UTM GRID) COORDINATES

5.3.1 The following formulae provide the only direct method for obtaining grid bearings and spheroidal distance from AMG (or UTM Grid) coordinates:

$\tan\theta_1 = (E_2' - E_1')/(N_2 - N_1)$ OR $\cot\theta_1 = (N_2 - N_1)/(E_2' - E_1')$

$L = (E_2' - E_1')/\sin\theta_1 = (N_2 - N_1)/\cos\theta_1$

$K = k_0 \left\{ 1 + [(E_1'^2 + E_1' E_2' + E_2'^2)/6r_m^2]\ \underline{[1 + (E_1'^2 + E_1' E_2' + E_2'^2)/36r_m^2]} \right\}$

$s = L/K$

$*\delta_1'' = -(N_2 - N_1)(E_2' + 2E_1')\ \underline{[1 - (E_2' + 2E_1')^2/27r_m^2]}/6r_m^2 \sin 1''$

$*\delta_2'' = (N_2 - N_1)(2E_2' + E_1')\ \underline{[1 - (2E_2' + E_1')^2/27r_m^2]}/6r_m^2 \sin 1''$

$\beta_1 = \theta_1 - \delta_1$

$\beta_2 = \theta_1 \pm 180° - \delta_2$

*In order to simplify programming, these formulae may be used in the form:

$\sin\delta_1 = -(N_2 - N_1)(E_2' + 2E_1')\ \underline{[1 - (E_2' + 2E_1')^2/27r_m^2]}/6r_m^2$

$\sin\delta_2 = (N_2 - N_1)(2E_2' + E_1')\ \underline{[1 - (2E_2' + E_1')^2/27r_m^2]}/6r_m^2$

5.3.2 The numerical examples at paragraphs *5.4* and *5.5* omit the terms underlined as, for most practical purposes, their effects are negligible. On a line 100 kilometres long running north and south on a zone boundary the errors are 0·″08 and 0·25 ppm i.e. about 25 mm.

5.4 GRID BEARINGS AND SPHEROIDAL DISTANCE FROM AMG COORDINATES: NUMERICAL EXAMPLE, AUSTRALIAN NATIONAL SPHEROID

From: BUNINYONG E_1 = 228 742·077 N_1 = 5 828 074·208 Zone 55
False Origin = 500 000·000
E'_1 = - 271 257·923

To: FLINDERS PEAK E_2 = 273 629·436 N_2 = 5 796 305·236
False Origin = 500 000·000
E'_2 = - 226 370·564

Latitude functions:

Using the method suggested in *5.2.2*, with N_m = 5 812 189·72

ϕ'_m = - 37°·84
$1/6r_m^2$ = 4·107 06 . 10^{-15}
$1/6r_m^2 \sin 1''$ = 8·471 43 . 10^{-10}

Grid Bearings and Spheroidal Distance:

$(E'_2 - E'_1)$ = 44 887·359
$(N_2 - N_1)$ = - 31 768·972
$\cot\theta_1$ = $(N_2 - N_1)/(E'_2 - E'_1)$
= - 0·707 748 75
θ_1 = 125° 17′ 20·″05
L = $(E'_2 - E'_1)/\sin\theta_1$
= 54 992·204
K = $k_0 [1 + (E_1'^2 + E'_1 E'_2 + E_2'^2)/6r_m^2]$
= 0·999 6 {1 + [(- 271 257·9)² + (- 271 257·9) (- 226 370·6) + (- 226 370·6)²] 4·107 06 . 10^{-15}}
= 1·000 364 56
s = L/K
= **54 972·164**
δ''_1 = $-(N_2 - N_1)(E'_2 + 2E'_1)/6r_m^2 \sin 1''$
= - (- 31 769) [(- 226 370·6) + 2 (- 271 257·9)] 8·471 43 . 10^{-10}
= -20·″69
δ''_2 = $(N_2 - N_1)(2E'_2 + E'_1)/6r_m^2 \sin 1''$
= (- 31 769) [2 (- 226 370·6) + (- 271 257·9)] 8·471 43 . 10^{-10}
= 19·″48
β_1 = $\theta_1 - \delta_1$ = **125° 17′ 40·″74**
β_2 = $\theta_1 \pm 180° - \delta_2$ = **305°** 17′ 00·″57

5.5 GRID BEARINGS AND SPHEROIDAL DISTANCE FROM UTM GRID COORDINATES: NUMERICAL EXAMPLE, WORLD GEODETIC SYSTEM 1972 SPHEROID

From Station: "M" E_1 = 787 420·487 N_1 = 6 782 165·201 Zone 58

False Origin = 500 000·000

E'_1 = 287 420·487

To Station: "X" E_2 = 841 341·166 N_2 = 6 800 667·210

False Origin = 500 000·000

E'_2 = 341 341·166

Latitude functions:

Using the method suggested in *5.2.2*, with N_m = 6 791 416·205

ϕ'_m = −29°·01

$1/6r_m^2$ = 4·114 88 . 10^{-15}

$1/6r_m^2 \sin 1''$ = 8·487 55 . 10^{-10}

Grid Bearings and Spheroidal Distance:

$(E'_2 - E'_1)$ = + 53 920·679

$(N_2 - N_1)$ = + 18 502·009

$\cot \theta_1$ = $(N_2 - N_1)/(E'_2 - E'_1)$

= 0·343 133 83

θ_1 = 71° 03′ 40·″22

L = $(E'_2 - E'_1)/\sin \theta_1$

= 57 006·701

K = $k_0[1 + (E'^2_1 + E'_1 E'_2 + E'^2_2)/6r_m^2]$

= 0·999 6 $\{1 + [(287\,420{\cdot}5)^2 + (287\,420{\cdot}5)(341\,341{\cdot}2) + (341\,341{\cdot}2)^2]\, 4{\cdot}114\,88 \,.\, 10^{-15}\}$

= 1·000 822 59

s = L/K

= **56 959·846**

δ''_1 = $-(N_2 - N_1)(E'_2 + 2E'_1)/6r_m^2 \sin 1''$

= − 18 502 [341 341·2 + 2 (287 420·5)] 8·487 55 . 10^{-10}

= − 14·″39

δ''_2 = $(N_2 - N_1)(2E'_2 + E'_1)/6r_m^2 \sin 1''$

= 18 502 [2(341 341·2) + 287 420·5] 8·487 55 . 10^{-10}

= + 15·″23

β_1 = $\theta_1 - \delta_1$ = **71° 03′ 54·″61**

β_2 = $\theta_1 \pm 180° - \delta_2$ = **251° 03′ 24·″99**

5.6 AMG (OR UTM GRID) COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE

5.6.1 This computation is commonly used by surveyors when computing on the grid. The coordinates of one station are known and the grid bearing and spheroidal distance from this station to an adjacent station are determined. These quantities are applied to the coordinates of the known station to derive the coordinates of the unknown station and the reverse grid bearing from this station to the known station. Refer to the traverse diagram at Figure 5.2.

The formulae given in paragraph *5.6.2* firstly require the computation of the approximate coordinates of the unknown station using

$$E'_2 \doteqdot E'_1 + k_1 \, s \sin \beta_1 \qquad E'_1 = E_1 - 500\,000$$

$$N_2 - N_1 \doteqdot k_1 \, s \cos \beta_1$$

If unknown, $k_1 \doteqdot 0{\cdot}999\,6 + 1{\cdot}23E'^2.10^{-14}$ (see *5.11.4*).

The value for the latitude function $1/6r^2$ is computed as already described in paragraph *5.2*, using

$$N_m = (N'_1 + N'_2)/2.$$

5.6.2 FORMULAE

$$K = k_0 \left\{ 1 + [(E'^2_1 + E'_1E'_2 + E'^2_2)/6r_m^2] \, \underline{[1 + (E'^2_1 + E'_1E'_2 + E'^2_2)/36r_m^2]} \right\}$$

$$L = sK$$

$$\sin \delta_1 = -(N_2 - N_1)(E'_2 + 2E'_1) \, \underline{[1 - (E'_2 + 2E'_1)^2/27r_m^2]}/6r_m^2$$

$$\theta = \beta_1 + \delta_1$$

$$\sin \delta_2 = (N_2 - N_1)(2E'_2 + E'_1) \, \underline{[1 - (2E'_2 + E'_1)^2/27r_m^2]}/6r_m^2$$

$$\beta_2 = \theta \pm 180^\circ - \delta_2$$

$$\Delta E = L \sin \theta \qquad E_2 = E_1 + \Delta E$$

$$\Delta N = L \cos \theta \qquad N_2 = N_1 + \Delta N$$

For lower order surveys the underlined terms are often omitted. The latitude function $1/6r^2$ becomes a constant and the formulae for K and δ are replaced by the simplified formulae found later in this chapter.

5.6.3 These formulae are accurate to 0·″02 and 0·1 ppm over any 100 kilometre line in an AMG (or UTM Grid) zone.

5.6.4 The numerical examples at paragraphs *5.7* and *5.8* omit the terms underlined as for most practical purposes their effects are negligible. For a line 100 kilometres long running north and south on a zone boundary, the errors are 0·″08 and 0·25 ppm (about 25 mm) respectively.

5.7 AMG COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE: NUMERICAL EXAMPLE, AUSTRALIAN NATIONAL SPHEROID

From: BUNINYONG E_1 = 228 742·077 N_1 = 5 828 074·208 Zone 55
False Origin = 500 000·000
E'_1 = - 271 257·923

To: FLINDERS PEAK Grid Bearing $\beta_1 = 125° 17' 40\cdot''72$
Spheroidal Distance s = 54 972·161 metres

$k_1 \doteq 1\cdot000\ 51$
$E'_2 \doteq E'_1 + k_1\, s \sin\beta_1$
$\doteq -226\ 367$
$N_2 - N_1 \doteq k_1\, s \cos\beta_1$
$\doteq -31\ 778$
$N_2 \doteq 5\ 796\ 296$ $N_m = 5\ 812\ 185$
$1/6r_m^2 = 4\cdot107\ 06 \,.\, 10^{-15}$ for N_m
$K = k_0\,[1 + (E'^2_1 + E'_1\, E'_2 + E'^2_2)/6r_m^2]$
$= 0\cdot999\ 6 \left\{1 + [(-271\ 257\cdot9)^2 + (-271\ 257\cdot9)\ (-226\ 367) + (-226\ 367)^2]\ 4\cdot107\ 06 \,.\, 10^{-15}\right\}$
$= 1\cdot000\ 364\ 5$
$L = 54\ 972\cdot161 \,.\, K = 54\ 992\cdot198$
$\sin\delta_1 = -(N_2 - N_1)\,(E'_2 + 2E'_1)/6r_m^2$
$= -(-31\ 778)\,[-226\ 367 + 2\,(-271\ 257\cdot9)]\ 4\cdot107\ 06 \,.\, 10^{-15}$
$\delta_1 = -20\cdot''7$
$\theta = \beta_1 + \delta_1 = 125° 17' 20\cdot''02$
$\sin\delta_2 = (N_2 - N_1)\,(2E'_2 + E'_1)/6r_m^2$
$= -31\ 778\,[2\,(-226\ 367) + (-271\ 257\cdot9)]\ 4\cdot107\ 06 \,.\, 10^{-15}$
$\delta_2 = 19\cdot''5$
$\beta_2 = \theta \pm 180 - \delta_2 = 305° 17' 00\cdot''5$
$\Delta E = L \sin\theta$ $\Delta N = L \cos\theta$
$= 44\ 887\cdot36$ $= -31\ 768\cdot96$

Buninyong	E_1 = 228 742·077	N_1 = 5 828 074·208
	= 44 887·36	= - 31 768·96
FLINDERS PEAK	E_2 = **273 629·44**	N_2 = **5 796 305·25**

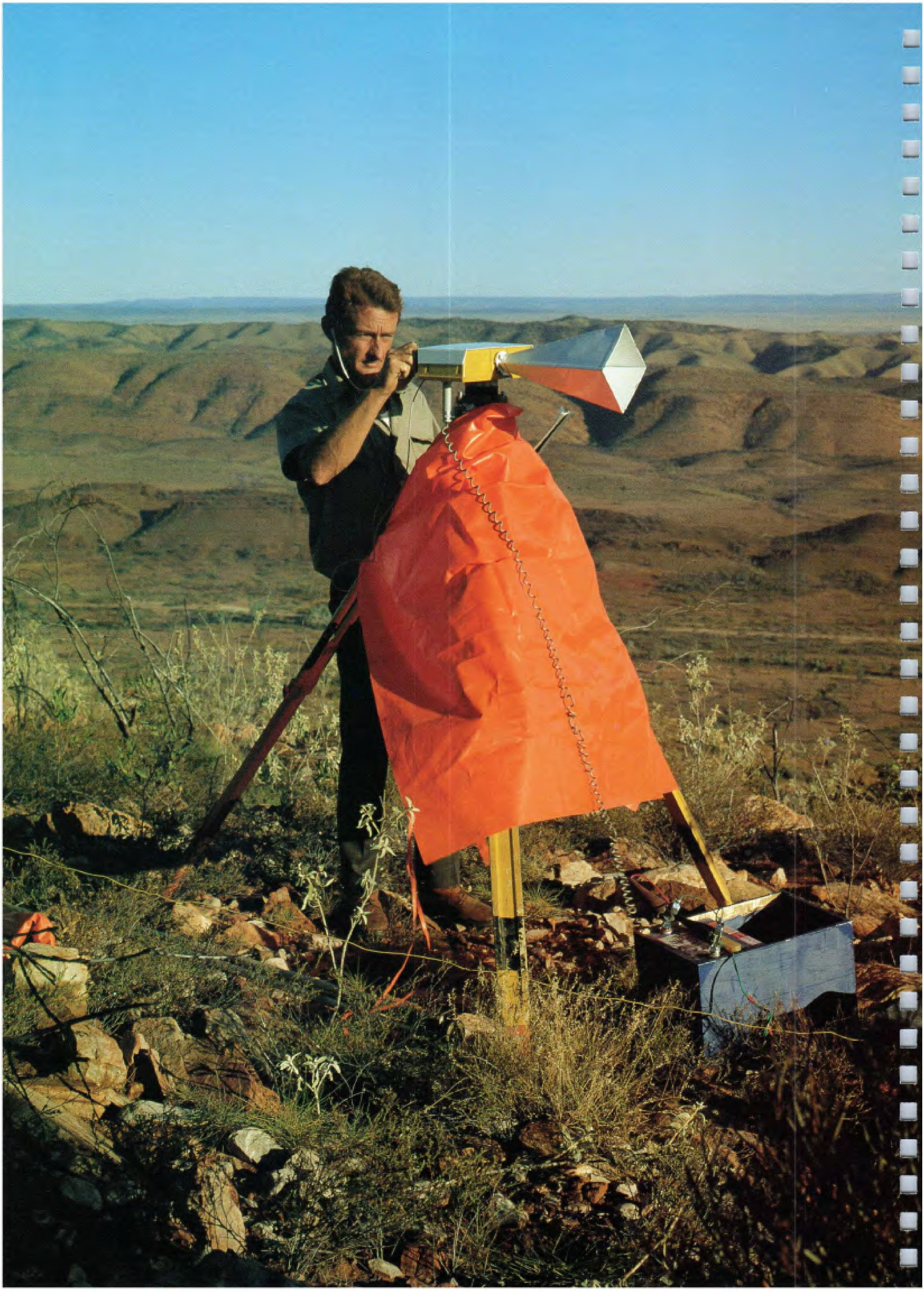
5.8 UTM GRID COORDINATES FROM GRID BEARING AND SPHEROIDAL DISTANCE: NUMERICAL EXAMPLE, WORLD GEODETIC SYSTEM 1972 SPHEROID

From: "M" E_1 = 787 420·487 N_1 = 6 782 165·201 Zone 58
False Origin = 500 000·000
E'_1 = 287 420·487

To: "X"
Grid Bearing $\beta_1 = 71°03'54\cdot''60$
Spheroidal Distance s = 56 959·840 metres

$k \doteq 1\cdot000\ 62$
$E'_2 \doteq E'_1 + k_1\, s \sin\beta_1$
$\doteq 341\ 331\cdot5$
$N_2 - N_1 \doteq k_1\, s \cos\beta_1$
$\doteq 18\ 494\cdot5$
$N_2 \doteq 6\ 800\ 660$ $N_m = 6\ 791\ 413$
$1/6r_m^2 = 4\cdot114\ 87 \,.\, 10^{-15}$ for N_m
$K = k_0[1 + (E'^2_1 + E'_1\, E'_2 + E'^2_2)/6r_m^2]$
$= 0\cdot999\ 6 \left\{1 + [(287\ 420\cdot5)^2 + (287\ 420\cdot5)\ (341\ 331) + (341\ 331)^2]\ 4\cdot114\ 87 \,.\, 10^{-15}\right\}$
$= 1\cdot000\ 822\ 55$
$L = 56\ 959\cdot840 \,.\, K = 57\ 006\cdot692$
$\sin\delta_1 = -(N_2 - N_1)\,(E'_2 + 2E'_1)/6r_m^2$
$= -18\ 494\cdot5\,[341\ 331\cdot5 + 2\,(287\ 420\cdot5)]\ 4\cdot114\ 87 \,.\, 10^{-15}$
$\delta_1 = -14\cdot''38$
$\theta = \beta_1 + \delta_1 = 71°03'40\cdot''22$
$\sin\delta_2 = (N_2 - N_1)\,(2E'_2 + E'_1)/6r_m^2$
$= 18\ 494\cdot5\,[\,2\,(341\ 331\cdot5) + 287\ 420\cdot5]\ 4\cdot114\ 87 \,.\, 10^{-15}$
$\delta_2 = 15\cdot''23$
$\beta_2 = \theta \pm 180° - \delta_2 = 251°03'\ 24\cdot''99$
$\Delta E = L \sin\theta$ $\Delta N = L \cos\theta$
$= 53\ 920\cdot67$ $= 18\ 502\cdot01$

Station "M"	E_1 = 787 420·487	N_1 = 6 782 165·201
	53 920·67	18 502·01
Station "X"	E_2 = **841 341·16**	N_2 = **6 800 667·21**



5.9 ZONE TO ZONE TRANSFORMATIONS

It may sometimes be necessary to compute the grid coordinates in a particular zone for a point whose grid coordinates are known in the adjacent zone. Any one of the following methods may be used:

(1) convert the known grid coordinates to latitude and longitude using Redfearn's formulae as given in paragraph *4.8*, and then convert back to grid coordinates in the adjacent zone again using Redfearn's formulae as given in paragraph *4.5*. Although this method of zone to zone transformation is lengthy, it can still be carried out on the larger programmable calculators;

(2) Lauf's method, as described in Chapter 6;

(3) where geographical coordinates are not required, zone to zone transformations on the Transverse Mercator projection may be directly computed using the following formulae adapted from Jordan-Eggert: *Handbuch der Vermessungskunde* (Volume III, second half-volume, section 38, 1941 edition). These formulae are accurate to 10 mm anywhere within half a degree of a zone boundary.

Formulae

$$E_2 = 500\,000 - E'_Z + (E'_1 - E'_Z)\cos 2\gamma_Z - (N_1 - N_Z)\sin 2\gamma_Z + H_1L^2 \sin(2\theta_Z + J_1)$$

$$N_2 = N_Z + (N_1 - N_Z)\cos 2\gamma_Z + (E'_1 - E'_Z)\sin 2\gamma_Z + H_1L^2 \cos(2\theta_Z + J_1)$$

where:

Z = a point on the zone boundary

E_1, N_1 = known coordinates of the point to be transformed

E_2, N_2 = required coordinates of the point in the adjacent zone

θ_Z = plane bearing from Z to the point to be transformed

$\tan J_1 = [\omega_Z^2 \cos^2\phi_Z (1 + 31 \tan^2\phi_Z) - 6(1 + e'^2 \cos^2\phi_Z)]/[18\omega_Z \sin\phi_Z (1 + e'^2 \cos^2\phi_Z)]$

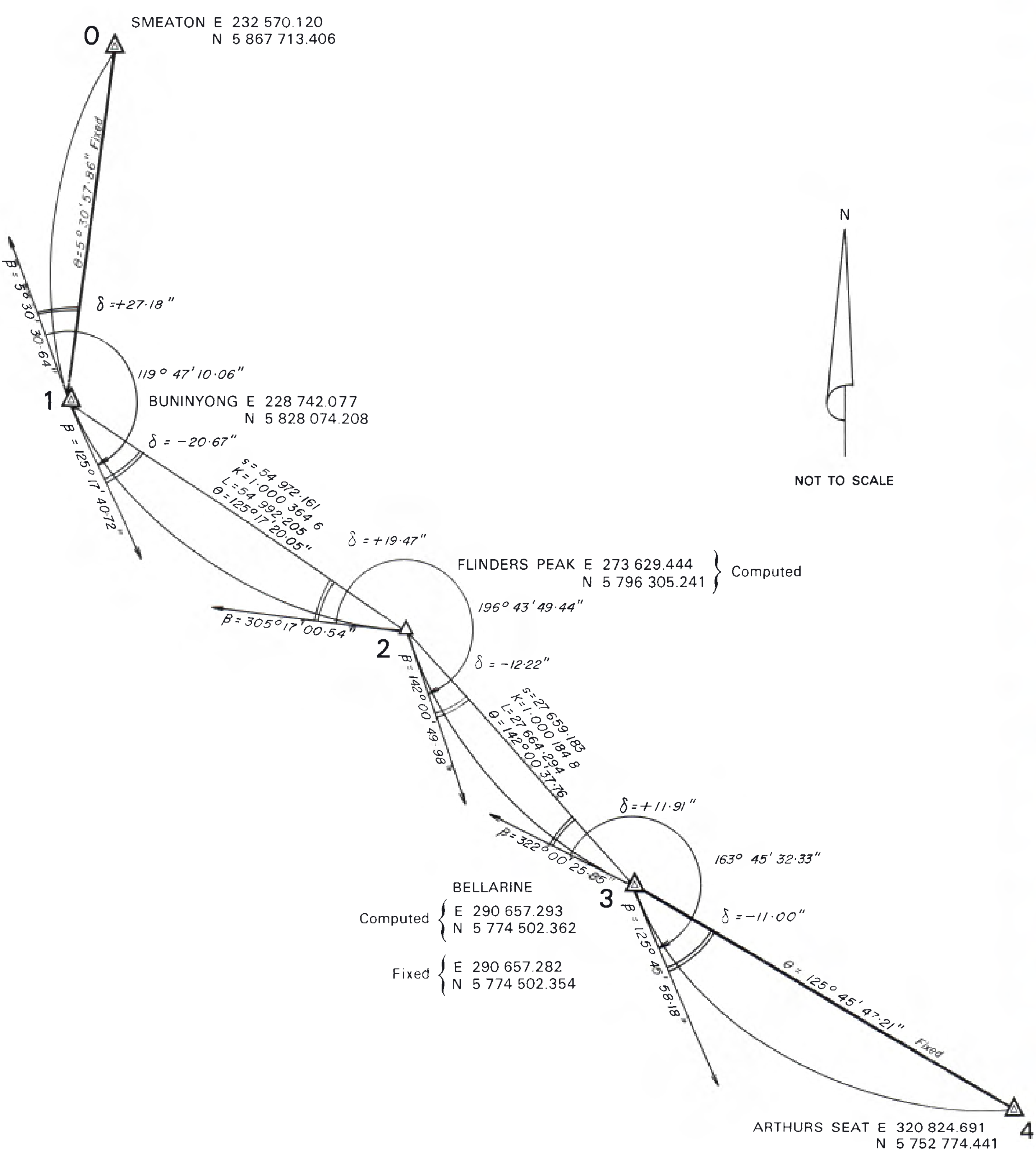
$H_1 = -3\omega_Z^2 \sin\phi_Z \cos\phi_Z / (\rho_Z \cos J_1)$

A similar method of direct zone to zone transformation, as proposed by Gotthardt, is published in the German textbook by Grossman, *Geodetische Rechnungen und Abbildungen*, 1964 edition, page 188.

CA 1000 Tellurometer measurements near Hermannsburg, Macdonnell Ranges, Northern Territory *Photo: B. M. Allwright*

Figure 5.1 DIAGRAM SHOWING TRAVERSE FROM BUNINYONG TO BELLARINE

Coordinates are shown in terms of AMG Zone 55



NOTE: The quantities θ, δ, β, K and L have been computed using the rigorous formulae

5.10 TRAVERSE COMPUTATION USING ARC-TO-CHORD CORRECTIONS AND LINE SCALE FACTORS

5.10.1 The method can be varied to suit the requirements of the job. At one extreme, the arc-to-chord corrections and line scale factors can be ignored and the traverse computed using the formulae of plane trigonometry; or, if good quality maps showing the AMG or UTM Grid are available, traverse stations may be plotted by inspection and the approximate coordinates scaled with sufficient precision to enable computation of the arc-to-chord corrections and line scale factors. At the other extreme, the arc-to-chord corrections and line scale factors can be computed precisely, and the method becomes first order anywhere in an AMG or UTM Grid zone. The precision obtained can be closely balanced against the labour involved. Arc-to-chord corrections rarely approach 100″ in magnitude, and corrections to lengths rarely attain 50 metres, so that even for the highest precision much of the work can be done with four significant figures. Prior to precise computation, approximate coordinates and bearings may be carried through the traverse, using uncorrected field measurements, to ensure that the observations are free of gross errors. A diagram of the traverse approximately to scale is desirable.

5.10.2 BASIC OUTLINE

There are many ways of arranging the computation. Essentially, the work is split into stages:

(1) approximate eastings and northings are computed from observed angles and distances;
(2) arc-to-chord corrections and line scale factors are computed from the approximate coordinates and applied to the observations to give plane angles and plane distances;
(3) precise coordinates are computed by plane trigonometry;
(4) misclosure in grid bearing and position is analysed and the traverse or figure adjusted as required.

For precise computation, each line is rigorously computed before the next line is calculated, so that errors in the approximate coordinates do not accumulate. True eastings E′ and differences in northing ΔN are the quantities carried through the computation. Sign conventions can be disregarded and signs determined by inspection of a traverse diagram, similar to that shown in Figure 5.2.

5.10.3 FORMULAE AND SYMBOLS

Formulae for arc-to-chord corrections and line scale factors are given at paragraph *5.6.2.* If the underlined terms are omitted, the errors for a 100 kilometre line running north and south on a zone boundary do not exceed 0·″08 in bearing and 0·25 ppm in distance. As the final coordinates of a precise traverse will nearly always be computed and adjusted by least squares on a computer, the underlined terms are rarely used in computations on a programmable calculator. For traverses of lower order, the simplified formulae given in paragraphs *5.11, 5.12* and *5.13* can be used. For short lines near a central meridian it may be possible to omit the arc-to-chord corrections and line scale factors and compute the traverse with observed angles and distances, using the formulae of plane trigonometry.

If the symbol δ_{21} is used for the arc-to-chord correction at station 2 to station 1 and δ_{23} for the correction at station 2 to station 3 and the angles are measured clockwise from station 1 to station 3, then the plane angle P_2 at station 2 is obtained from the observed angle 0_2 by:

$$P_2 = 0_2 + \delta_{23} - \delta_{21}$$

where angles are measured clockwise only.

5.10.4 COMPUTATIONS OF ARC-TO-CHORD CORRECTIONS AND SCALE FACTORS

Although there are several ways of arranging the computation, the following procedure, which can easily be performed on many types of programmable calculators, is recommended:

(1) compute the grid bearing to the "forward" station by applying the observed horizontal angle at the "occupied" station to the known grid bearing of the "rear" station;
(2) compute the point scale factor at the "occupied" station and multiply the spheroidal distance to the "forward" station by this factor;
(3) using the distance obtained and the forward grid bearing, compute approximate coordinates of the "forward" station by plane trigonometry;
(4) using the coordinates of the "occupied" station and the approximate coordinates of the "forward" station, compute the arc-to-chord correction at the "occupied" station and the line scale factor. If the line crosses the central meridian, (E'_1 E'_2) is negative;
(5) add the arc-to-chord correction to the forward grid bearing to obtain the plane bearing and multiply the spheroidal distance by the line scale factor to obtain the plane distance;
(6) using the plane bearing and plane distance, compute the coordinates of the "forward" station by plane trigonometry;
(7) compute the arc-to-chord correction from the new station to the previously occupied station and add this to the plane bearing reversed by 180° to obtain the reverse grid bearing from the new station.

The above process is repeated for each new line of a traverse with the reverse grid bearing of the previous line becoming the known grid bearing to the rear station.

The traverse diagram shown in Figure 5.2 should be referred to with the above text.

5.11 SIMPLIFIED FORMULAE FOR SCALE FACTORS

5.11.1 In order to convert a spheroidal distance, s, to a plane distance, L, or to obtain the spheroidal distance from a plane or grid distance, it is necessary to calculate the line scale factor. Since a Transverse Mercator projection is conformal, the scale at any point is the same in all directions, but varies with the distance from the central meridian.

5.11.2 To minimise the scale factor over the whole of the zone width, a central scale factor k_0 = 0·999 6 is superimposed over all projected distances. After the application of this central scale factor, the point scale factor will vary from 0·999 6 on a central meridian to almost 1·001 0 on the zone boundary at latitude 8° South and to 1·000 3 on a zone boundary at latitude 45° South for the Australian Map Grid. For the UTM Grid (WGS 72 spheroid) the point scale factors will vary between the same values between the equator and latitude 56° South.

5.11.3 POINT SCALE FACTOR

The rigorous equation for point scale factor given in paragraph *4.5.4* can be reduced to:

$$k = k_0[1 + (E'^2/2r_m^2) + (E'^4/24r_m^4)]$$

This equation is accurate to 1 part in 10 million. Omission of the last term results in an accuracy of 2 parts in 10 milion.

5.11.4 The equation can be simplified further using the following approximation:

$$k = 0{\cdot}999\,6 + 1{\cdot}23\ E'^2\ 10^{-14}$$

which is easy to remember. The formula is correct at latitude 34° and is accurate to 8 ppm at the equator.

Examples:

AMG

Station: BUNINYONG, Zone 55

$E' = -271\ 300$

$k = 0{\cdot}999\,6 + 1{\cdot}23\,[(-271\ 300)^2.10^{-14}]$

$= 1{\cdot}000\ 505$

which is accurate to 2 ppm.

UTM Grid

Station: "M", Zone 58

$E' = 287\ 400$

$k = 0{\cdot}999\,6 + 1{\cdot}23[(287\ 400)^2.10^{-14}]$

$= 1{\cdot}000\ 616$

which is accurate to 4 ppm.

Point scale factors may be taken from the nomogram in Figure 5.4. For surveys of lower acuracy and of limited extent, the point scale factor can be substituted for the line scale factor and combined with the height factor to reduce slope distances for use in AMG or UTM Grid computations. See paragraphs *5.11.7* and *5.13*.

5.11.5 LINE SCALE FACTOR

The scale factor will in general vary from one end of a line to the other, and the method used to determine a scale factor for the whole line will depend upon the accuracy of the survey and the length of the line.

5.11.6 The equation for line scale factor given in paragraph *5.6.2* can be reduced to:

$$K = k_0\,[1 + (E_1'^2 + E_1'E_2' + E_2'^2)/6r_m^2]$$

5.11.7 For single lines or traverses, line scale factors can be obtained anywhere on the AMG (or UTM Grid) by using either:

(1) the point scale factor for the mean easting of the line or traverse. This procedure is accurate to 1 ppm in any line or traverse extending 33 kilometres in easting;

OR

(2) the mean of the point scale factors at the extremities of the line or traverse. This procedure is accurate to 1 ppm in any line or traverse extending 16 kilometres in easting.

The accuracies stated above are independent of the location in the zone.

The technique explained in paragraph *5.10.4* can be used with this method of obtaining line scale factor.

5.12 SIMPLIFIED FORMULAE FOR ARC-TO-CHORD CORRECTIONS

5.12.1 On the zone boundary the arc-to-chord correction for a north-south line is about 0·″85 per kilometre.

The formula for arc-to-chord correction given in paragraph *5.6.2*, after omitting the underlined term, can be written:

$$\delta_1'' = -(N_2 - N_1)(2E_1 + E_2 - 1{\cdot}5.10^6)/6r_m^2 \sin 1''$$

This yields results correct to 0.″01 for a line 30 kilometres long.

For programming aplications, this formula may be written as follows:

$$\sin \delta_1 = -(N_2 - N_1)(2E_1 + E_2 - 1{\cdot}5.10^6)/6r_m^2$$

5.12.2 If the numerical difference between the arc-to-chord corrections at each end of the line can be neglected, the formula can be simplified to:

$$\delta_1'' = -\delta_2'' = -(N_2 - N_1)(E_2 + E_1 - 10^6)/4r_m^2 \sin 1''$$
$$= \Delta\beta/2$$

5.12.3 The function $1/4r_m^2 \sin 1''$ has a value of 0·127.10⁻⁸ anywhere on the Australian Map Grid and on the UTM Grid (WGS 72 spheroid) between 8° and 57° South latitude. Using this value, the formula can be reduced to:

$$\delta_1'' = -\delta_2'' = -(N_2 - N_1)(E_2 + E_1 - 10^6)(0{\cdot}127.10^{-8})$$

The error in this simplified formula is 0·″6 on the test line Buninyong-Flinders Peak.

Alternatively, arc-to-chord corrections can be read off the nomogram in Figure 5.3.

5.12.4 The sign of the arc-to-chord correction will always follow strictly from the given formula but can be quickly checked by reference to a traverse diagram. However, some care is needed in the case where a line crosses the central meridian.

On an orthomorphic grid such as the AMG or the UTM Grid, the geodesic joining two points will always plot as a curved line lying on that side of the straight line joining the two points where the projection scale factor is greater. Therefore, the curvature of this line will be reversed at the point where the line crosses the central meridian. However, in the case where the central meridian divides a line in such a way that one part of the line is less than one third of the total line length, the diagram approach for determination of the sign of the arc-to-chord correction will fail. In this case the sign is determined by the concavity of the longer part.

The diagram below, which is greatly exaggerated, illustrates this special case.

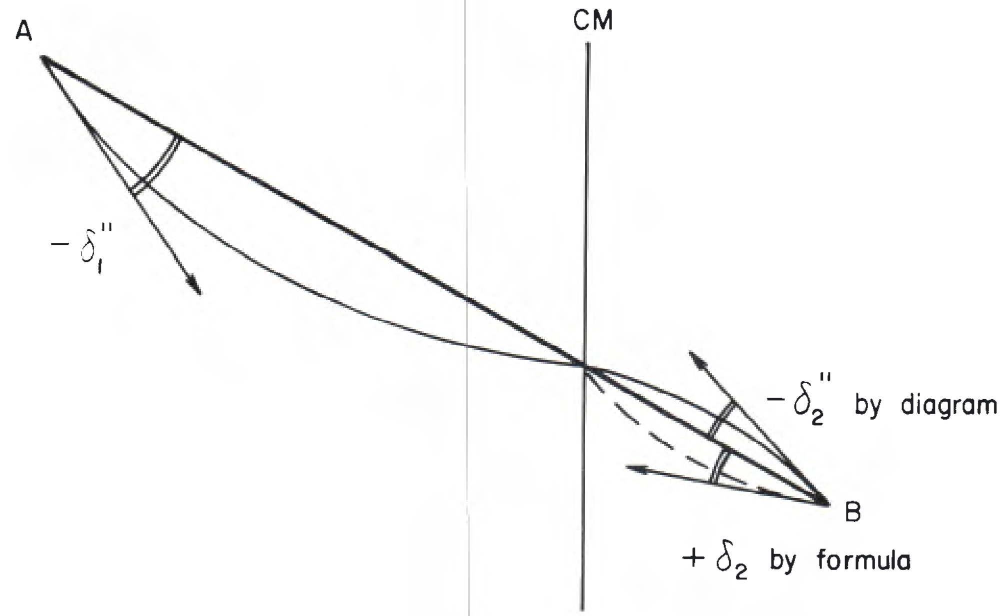


Figure 5.2

5.13 COMBINED POINT SCALE AND HEIGHT FACTOR

5.13.1 For surveys of lower accuracy and of limited extent, it may be quite adequate to obtain the line scale factor by using the methods already given in paragraph *5.11.7*, or by simply deriving a "locality" scale factor based on the mean easting of the centre of the survey area. This factor can then be combined with a "line" or "locality" height factor, as determined in paragraph *2.3.5*, to obtain grid (plane) distances for computation of coordinates on the AMG or UTM Grid. The procedure for the rigorous reduction of measured distances to the surface of the spheroid has already been covered in paragraph *2.3.2*.

5.13.2 The combined point scale and height factor is simply obtained by multiplying the point or locality scale factor by the line or locality height factor.

Numerical Example

s = 3 264·16 metres
h_m = 400 metres
ϕ_m = 35° $\quad$ R_m = 6 370 800 (from table in 2.3.5)
E_m = 228 700 $\quad$ E'_m = - 271 300

Point scale factor, $k = 0{\cdot}999\,6 + 1{\cdot}23.E'^2.10^{-14}$
$= \mathbf{1{\cdot}000\,505}$

Height Factor $= 1 - \dfrac{400}{6\,370\,800 + 400}$
$= \mathbf{0{\cdot}999\,937}$

Combined point scale and height factor = 1·000 505 (0·999 937)
= **1·000 442**

Grid (plane) distance = 3 264·16 (1·000 442)
= **3 265·60 metres**

A nomogram can be prepared to give the combined point scale and height factor for any given region.

5.14 SIMPLIFIED FORMULAE FOR GRID CONVERGENCE

5.14.1 When performing simplified computations on the AMG or UTM Grid it may occasionally be required to convert an azimuth to a grid bearing. A grid bearing, β, is derived from an azimuth, α, by algebraically adding the grid convergence, γ, to the azimuth. In the southern hemisphere, grid convergence is positive east of the central meridian.

$$\beta = \alpha + \gamma$$

5.14.2 Grid convergence is most easily determined from geographical coordinates. The rigorous formula given in paragraph *4.5.3* for obtaining grid convergence from geographical coordinates may be reduced to:

for angles in seconds of arc

$$* \; \gamma'' = \omega'' \sin\phi + \frac{1}{3(206\ 265)^2} (\omega'')^3 \sin\phi \cos^2\phi \, (2\psi^2 - \psi) + \ldots$$

This formula may be further reduced to:

$$\tan\gamma = - \sin\phi \tan\omega$$

which is accurate to less than 0·"1 on both of the test lines.

Depending upon the accuracy required, values for ϕ and ω (= $\lambda - \lambda_0$) may be taken from any good quality map.

5.14.3 Alternatively, the rigorous formula for obtaining grid convergence from AMG or UTM Grid coordinates given in paragraph *4.8.3* may be reduced to:

for angles in seconds of arc

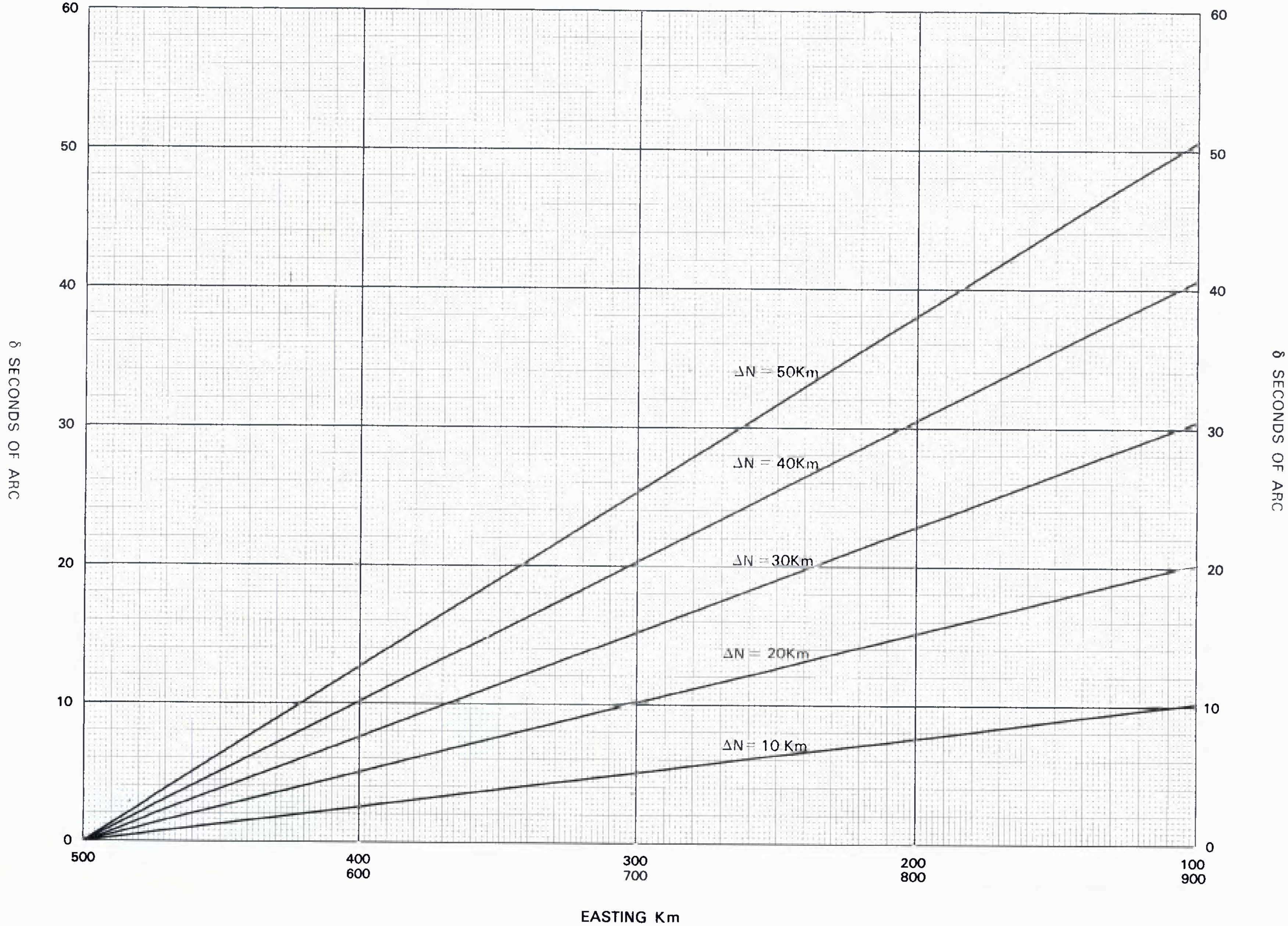
$$* \; \gamma'' = [- t'x + t' \frac{(x^3)}{3} (- 2\psi'^2 + 3\psi' + t'^2)] \; 206\ 265$$

where $x = E'/k_0\nu'$

* Adapted from *Geodesy and Map Projections* by Professor G.B. Lauf and reproduced by permission of the Royal Melbourne Institute of Technology Ltd.

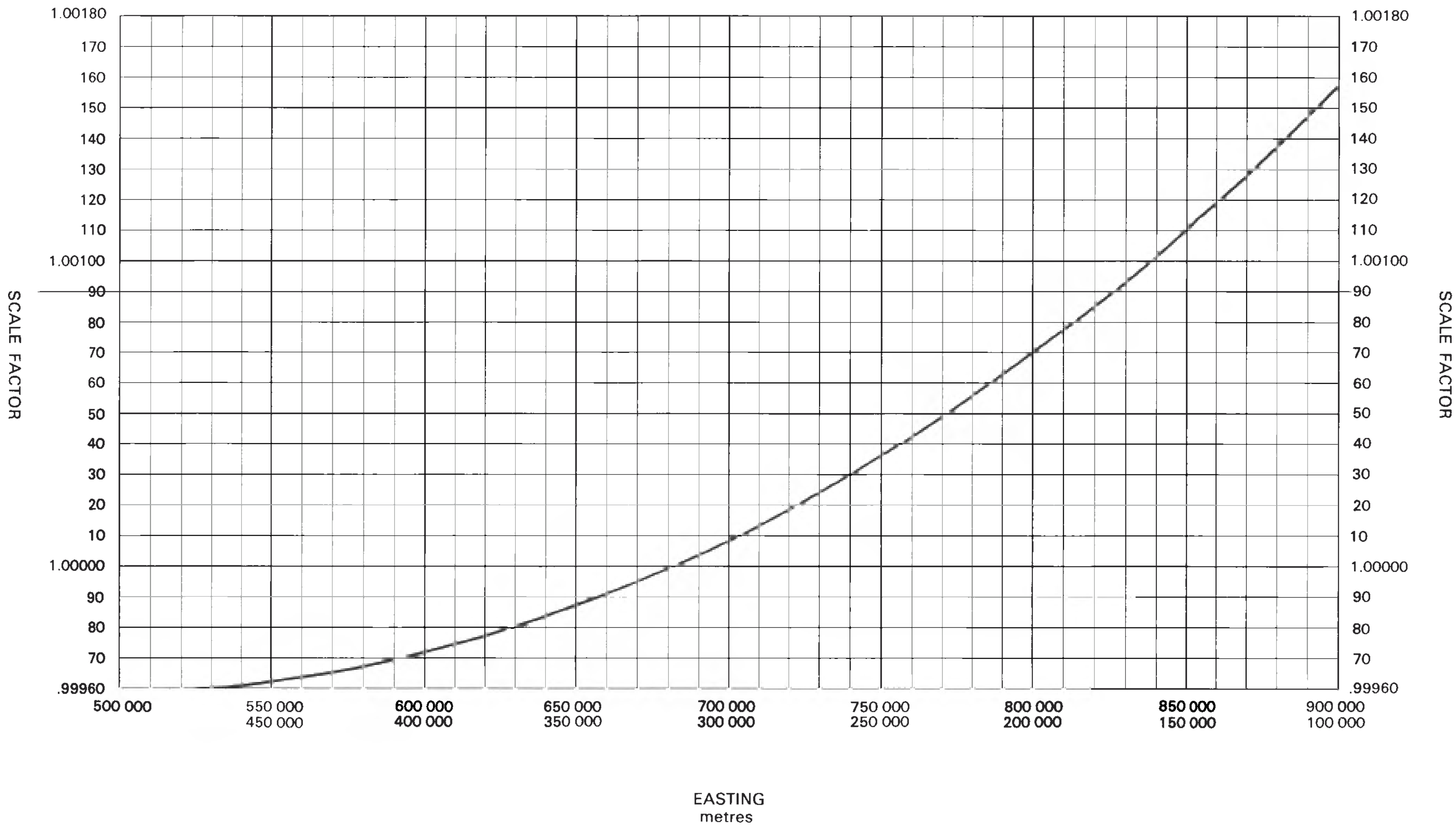
Figure 5.3 AUSTRALIAN MAP GRID AND UNIVERSAL TRANSVERSE MERCATOR GRID

ARC - TO - CHORD CORRECTION δ



NOTE: This nomogram has been compiled for latitude 26° South. For a line of ΔN=50 kilometres lying in either latitude 8° or 45° South, the nomogram is accurate to 0.1″ and 0.2″ respectively

Figure 5.4 AUSTRALIAN MAP GRID AND UNIVERSAL TRANSVERSE MERCATOR GRID POINT SCALE FACTOR



6.

TRANSFORMATION OF COORDINATES FROM ONE MAP PROJECTION TO ANOTHER

6.1 AIMS

6.1.1 The aim of this chapter is to provide formulae and a numerical example for the transformation of coordinates known on one map projection to coordinates on another projection using the method described by Professor G.B. Lauf.

6.1.2 The method given is only applicable where both coordinate systems are based on orthomorphic projections on the same reference spheroid. However, it can be used to transform pre-1966 rectangular coordinate systems onto the Australian Map Grid without the introduction of significant errors. For surveys of lower accuracy, the method may also be used to transform between the AMG66 and AMG84 coordinate sets, or vice-versa.

6.1.3 A detailed description of the method is given in the *Bulletin Géodésique*, Vol. 61, September 1961, under the title "Conformal Transformation from One Map Projection to Another, Using Divided Difference Interpolation".

6.2 USE OF THE CLARKE 1858 REFERENCE SPHEROID IN CONJUNCTION WITH THE AUSTRALIAN NATIONAL GRID

6.2.1 Prior to the introduction of the Australian Geodetic Datum and the Australian Map Grid in 1966, the Clarke 1858 reference spheroid was in widespread use throughout Australia. Latitudes and longitudes were computed on this spheroid in terms of the various State and local coordinate origins then in use. The rectangular grid coordinate system used in conjunction with this spheroid was called the Australian National Grid. Coordinates were quoted in yards and were derived from a Transverse Mercator projection of latitudes and longitudes as determined in relation to the relevant State or local coordinate origin then in use.

6.2.2 Apart from the basic differences in the reference spheroid and the unit of length used, the Australian National Grid differed significantly from the present Australian Map Grid in zone width and numbering, true and false coordinate origins and central scale factor.

6.2.3 Due to the number of State and local coordinate origins in use before the introduction of the Australian Map Grid in 1966, it is not possible to provide a single set of parameters which will enable transformation from the Australian National Grid to the Australian Map Grid.

6.2.4 If there is a sufficient number of points whose coordinates are known in both coordinate systems, then it may be possible to construct a diagram showing contours of difference between both systems. However, a more rigorous approach is to use these same common points in conjunction with Lauf's method.

Whichever method is used, care must be taken to ensure that the points to be transformed are interpolated and not extrapolated.

6.3 *LAUF'S METHOD

Coordinates are represented by a complex number

$Z = Y + iX$ (New grid)
$z = y + ix$ (Old grid)

where X,x = northing
Y,y = easting

Set up a table of divided differences such that

First order

$$\Delta Z_1 = \frac{Z_2 - Z_1}{z_2 - z_1}$$

$$\Delta Z_2 = \frac{Z_3 - Z_2}{z_3 - z_2}$$

$$\Delta Z_3 = \frac{Z_4 - Z_3}{z_4 - z_3}$$

Second order

$$\Delta^2 Z_1 = \frac{\Delta Z_2 - \Delta Z_1}{z_3 - z_1}$$

$$\Delta^2 Z_2 = \frac{\Delta Z_3 - \Delta Z_2}{z_4 - z_2}$$

Third order

$$\Delta^3 Z_1 = \frac{\Delta^2 Z_2 - \Delta^2 Z_1}{z_4 - z_1}$$

Common Points	*Old System*	*New System*	*Divided Differences* *First*	*Second*	*Third*
P_1	$z_1 = y_1+ix_1$	$Z_1=Y_1+iX_1$			
			ΔZ_1		
P_2	$z_2 = y_2+ix_2$	$Z_2=Y_2+iX_2$		$\Delta^2 Z_1$	
			ΔZ_2		$\Delta^3 Z_1$
P_3	$z_3 = y_3+ix_3$	$Z_3=Y_3+iX_3$		$\Delta^2 Z_2$	
			ΔZ_3		
P_4	$z_4 = y_4+ix_4$	$Z_4=Y_4+iX_4$			

For the point to be transformed (z), the divided differences are

$$\Delta Z = \frac{Z_1 - Z}{z_1 - z}$$

$$\Delta^2 Z = \frac{\Delta Z_1 - \Delta Z}{z_2 - z}$$

$$\Delta^3 Z = \frac{\Delta^2 Z_2 - \Delta^2 Z}{z_3 - z}$$

and the coordinates in the new system are—

$$Z = Z_1 + (z - z_1)\,\Delta Z_1 + (z - z_1)(z - z_2)\,\Delta^2 Z_1 + (z - z_1)(z - z_2)(z - z_3)\,\Delta^3 Z$$

For conformal map projections, with points less than a few hundred kilometres apart, we can substitute—

$$\Delta^3 Z = \Delta^3 Z_1$$

and the formula becomes—

$$Z = Z_1 + (z - z_1)\,\Delta Z_1 + (z - z_1)(z - z_2)\,\Delta^2 Z_1 + (z - z_1)(z - z_2)(z - z_3)\,\Delta^3 Z_1$$

As a check on the results, interpolate upwards from the bottom of the table and find—

$$Z = Z_4 + (z - z_4)\,\Delta Z_3 + (z - z_4)(z - z_3)\,\Delta^2 Z_2 + (z - z_4)(z - z_3)(z - z_2)\,\Delta^3 Z_1$$

6.4 FROM AUSTRALIAN NATIONAL GRID TO AUSTRALIAN MAP GRID : NUMERICAL EXAMPLE

In this example only intermediate results, which should be adequate for checking computer or calculator programs, are given.

		AUSTRALIAN NATIONAL GRID				*AUSTRALIAN MAP GRID (ZONE 54)*			
		z		$z_n - z_{n-1}$		Z		$Z_n - Z_{n-1}$	
n	*Name*	*East*	*North*	*East*	*North*	*East*	*North*	*East*	*North*
1	Misery	167 176·0	384 895·8			729 628·000	5 857 987·856		
				37 407·3	12 738·6			34 770·399	9 821·825
2	Smeaton	204 583·3	397 634·4			764 398·399	5 867 809·681		
				-15 926·0	-114 085·9			-20 112·343	-103 393·114
3	Gellibrand	188 657·3	283 548·5			744 286·056	5 764 416·567		
				44 774·1	43 215·7			43 011·245	37 263·415
4	Anakie	233 431·4	326 764·2			787 297·301	5 801 679·982		

Point to be transformed (z) : BUNINYONG
= 199 935·1 East 354 337·0 North

DIVIDED DIFFERENCES

	First $\Delta^1 Z$		*Second* $\Delta^2 Z$		*Third* $\Delta^3 Z$	
1	0·913 039 903	- 0·048 360 216				
			$4{\cdot}508 \cdot 10^{-9}$	$-4{\cdot}580 \cdot 10^{-10}$		
2	0·913 090 329	- 0·048 826 949			$-1{\cdot}342 \cdot 10^{-15}$	$-4{\cdot}064 \cdot 10^{-16}$
			$4{\cdot}396 \cdot 10^{-9}$	$-4{\cdot}069 \cdot 10^{-10}$		
3	0·913 188 298	- 0·049 150 212				

$z - z_1$	32 759·1	- 30 558·8
$z - z_2$	- 4 648·2	- 43 297·4
$z - z_3$	11 277·8	70 788·5

		$z - z_3$	$z - z_2$	$z - z_1$	ΔZ			
2nd Term	y			32 759·1	0·913 039 903		Z_1 729 628·000	5 857 987·856
	x			- 30 558·1	- 0·048 360 216	=	+28 432·535	-29 485·641
3rd Term	y		- 4 648·2	32 759·1	$4{\cdot}508 \cdot 10^{-9}$			
	x		- 43 297·4	- 30 558·1	$-4{\cdot}580 \cdot 10^{-10}$	=	- 7·236	- 5·078
4th Term	y	11 277·8	- 4 648·2	32 759·1	$-1{\cdot}342 \cdot 10^{-15}$			
	x	70 788·5	- 43 297·4	- 30 558·8	$-4{\cdot}064 \cdot 10^{-16}$	=	- 0·147	+ 0·129

SUM = **E 758 053·152** **N 5 828 497·266**
(AMG ZONE 54)

7.

GRID REFERENCES — MEDIUM AND LARGE SCALE MAPS

7.1 INTRODUCTION

7.1.1 The Universal Transverse Mercator (UTM) Grid, of which the Australian Map Grid (AMG) is part, covers the whole world except the polar regions beyond latitudes 84° N and 80° S. These areas are covered by the Universal Polar Stereographic (UPS) Grid (see paragraph *1.7.4*). Discussion of the UPS Grid is outside the scope of this manual.

7.1.2 Coordinates on the AMG are derived from a Transverse Mercator projection of latitudes and longitudes on the Australian Geodetic Datum. The same projection of the WGS 72 datum and spheroid is used to determine grid coordinates for Australian offshore islands and external territories lying outside the limits of the AMG (see paragraph *1.5*).

7.1.3 Grid coordinates on any given datum and grid may be used to uniquely define any point on the earth's surface. The grid zone is usually given first, followed by the eastings, and then the northings, both if necessary given to millimetres. Thus for survey station 'The Lion' we might find the AMG coordinates listed as:

Station	*Zone*	*Eastings*	*Northings*
LION	56	497 345·431	6 852 369·368

7.2 GRID REFERENCES

7.2.1 When describing the location of topographic features on a map, grid references may be used. Grid references usually consist of four or six digits, and are essentially generalised forms of grid coordinates. The first half of the total number of digits represents the easting, and the second half the northing.

7.2.2 To assist in determining grid references, maps at scales of 1:250 000 and larger show:

- a series of straight grid lines covering the map in squares—see Annex B, page 1. Maps at scales of 1:100 000 and larger generally have grid lines spaced at intervals of one kilometre. At 1:250 000 the grid lines are at intervals of 10 kilometres;
- an explanation in the margin of how to give a grid reference—see Annex B;
- a box in the margin showing the 1:100 000 map sheet numbers (not on all 1:250 000 series maps).

7.2.3 Examples of determining four and six figure grid references are shown at Annex B. When estimating the tenths part between grid lines, always round down. Thus a six figure grid reference defines a point as being within a 100 metre square, and a four figure grid reference, a 1 kilometre square. For the sample point, 'The Lion', the four and six figure grid references are 9752 and 973523 respectively.

7.2.4 A four figure grid reference is used on a 1:250 000 scale map. It may also be used on a larger scale map to describe an area feature such as a lake or city. In other cases, six figure grid references are generally used.

7.2.5 As grid references are a generalised form of grid coordinates, points with identical grid references will recur every 100 kilometres in easting or northing. For a unique reference, additional letters and figures are used. This is explained further in *7.3*.

7.3 UNIVERSAL GRID REFERENCES

7.3.1 Each zone of the AMG or UTM Grid is divided into 100 kilometre squares and each 100 kilometre square is given a pair of identifying letters. The letters for squares lying either wholly or partly within the area covered by the AMG are shown diagrammatically in Annex C. An explanation of the conventions used in determining the letters is also given. These conventions apply equally to the UTM Grid in general.

7.3.2 'The Lion' can be seen to lie within the 100 kilometre square having the letters MP. Thus a more specific reference for 'The Lion' is MP973523.

7.3.3 A single map sheet may contain parts of two or even four different 100 kilometre squares. The letters for each part are clearly indicated in the grid reference box, and are also indicated on the face of the map near the boundaries of 100 kilometre squares.

7.3.4 Due to the difference between the Australian Geodetic Datum and the WGS 72 datum, and also to differences between the defining parameters of the respective spheroids, it will be found that corresponding 100 kilometre square boundaries as generated from either geodetic datum will not coincide along the spheroid junction defined by the limits of the AMG. This will be exhibited as a shift between corresponding 100 kilometre square boundaries on maps published at 1:250 000 scale and larger which abut the spheroid junction.

7.3.5 For a unique grid reference, a further group of one or two figures and a letter is used. This is called the *Grid Zone Designation*. The figures are simply the 6° zone number as explained in paragraphs *1.3.2* and *1.7.3*. Some departures from this standard zone width occur north of latitude 56° N. Each zone is divided into rows which start at 80° S and proceed northerly to 84° N. The rows are lettered alphabetically C through X with the letters I and O omitted. Row X spans 12° of latitude, the remainder spanning 8° each. The Grid Zone Designation precedes the 100 kilometre square identification.

7.3.6 For 'The Lion', the Grid Zone Designation is 56J. Thus the complete grid reference for 'The Lion' is 56JMP973523.

7.3.7 Annex C shows the Grid Zone Designations within the area covered by the AMG. The Grid Zone Designation for a map is also indicated in the grid reference box in the margin of the map. Examples are shown in Annex B.

7.3.8 A universal grid reference is written without spaces or punctuation.

7.3.9 All elements of a universal grid reference need not be used. For brevity, and where confusion is unlikely to occur, the Grid Zone Designation and perhaps also the 100 kilometre square identification may be omitted.

7.3.10 A system preferred by some users is to replace the Grid Zone Designation with the number and name of the standard 1:100 000 map sheet on which the point falls. For example 'The Lion', which is on 1:100 000 map sheet 9441 Mount Lindesay, would be identified by:

9441 Mount Lindesay MP973523

The 100 kilometre square identifier may also be omitted without detriment. When using maps at larger scales, it is not necessary to quote the subdivision of the 1:100 000 sheet.

8.

THE AUSTRALIAN HEIGHT DATUM

8.1 INTRODUCTION

8.1.1 On 5 May 1971 the Division of National Mapping, on behalf of the National Mapping Council of Australia, carried out a simultaneous adjustment of 97 230 kilometres of two-way levelling. Mean sea level for 1966-1968 was assigned the value of zero on the Australian Height Datum at thirty tide gauges around the coast of the Australian continent.

8.1.2 The resulting datum surface, with minor modifications in two metropolitan areas, has been termed the *Australian Height Datum* (AHD) and was adopted by the National Mapping Council at its twenty-ninth meeting in May 1971 as the datum to which all vertical control for mapping is to be referred. The datum surface is that which passes through mean sea level at the thirty tide gauges and through points at zero AHD height vertically below the other basic junction points.

Further information on the determination of the AHD is given in Division of National Mapping Technical Report No. 12, *The Adjustment of the Australian Levelling Survey, 1970-71* (2nd edition, 1975).

8.1.3 The levelling network in Tasmania was adjusted on 17 October 1983 to re-establish heights on the *Australian Height Datum (Tasmania)*. This network, which consists of seventy-two sections between fifty-seven junction points is based on mean sea level for 1972 at the tide gauges at Hobart and Burnie. Mean sea level at both Hobart and Burnie was assigned the value of zero on the AHD (Tasmania).

8.2 BASIC AND SUPPLEMENTARY LEVELLING

8.2.1 Two-way levelling of third order accuracy or better, used in the original adjustment of 5 May 1971 which formed the AHD, is called "Basic Levelling". Levelling subsequently adjusted to the AHD is called "Supplementary Levelling".

8.2.2 Enquiries regarding details of basic and supplementary levelling in a State or Territory should be directed to the appropriate authority named in Annex F.

8.3 METROPOLITAN AND BUFFER ZONES

8.3.1 METROPOLITAN ZONES

Bench marks within the metropolitan areas of Perth and Adelaide are held fixed at heights assigned by the Surveyors General of Western Australia and South Australia. The areas in which these heights have been held fixed are termed "Metropolitan Zones".

The assigned heights within the Perth Metropolitan Zone are based on mean sea level at Fremantle over a different epoch from that used in the adjustment of 5 May 1971. These heights differed by not more than 4 mm from those computed in the adjustment.

The assigned heights within the Adelaide Metropolitan Zone are based on mean sea level at Port Adelaide and these heights differ by not more than 40 mm from those determined in the National Levelling Adjustment of 5 May 1971.

8.3.2 BUFFER ZONES

The small differences between the heights determined by the adjustment of 5 May 1971 and those assigned by the Surveyors General to bench marks on the perimeter of the Metropolitan Zones have been distributed through "Buffer Zones".

8.3.3 DELINEATION OF ZONES

Details relating to the limits of the Metropolitan and Buffer Zones, and the levelling sections within these zones may be obtained from the respective State Surveyors General.

8.3.4 HEIGHTS IN METROPOLITAN AND BUFFER ZONES

The heights of bench marks in the Metropolitan Zones assigned by the Surveyors General and the adjusted heights of bench marks in the Buffer Zones shall be regarded as being on the Australian Height Datum.

8.4 JUNCTION POINTS

8.4.1 Details relating to the localities, bench mark numbers, latitudes and longitudes and heights of the junction points in the basic levelling as adjusted on 5 May 1971 or as subsequently amended, may be obtained from the respective State Surveyors General or from the Division of National Mapping.

8.4.2 Diagrams showing the basic levelling, tide gauges and junction point numbers relating to the Australian Height Datum and the Australian Height Datum (Tasmania) are attached at Annex D.

8.5 FUTURE ADJUSTMENTS

8.5.1 Heights for any new bench marks within the Perth and Adelaide Metropolitan Zones will be assigned by the respective Surveyors General.

8.5.2 All new levelling of geodetic value outside these Metropolitan Zones shall be adjusted to the Australian Height Datum either by the Division of National Mapping or by the appropriate Council member, as may be agreed, using computer programs giving identical results, on the same format, and utilising the same junction point identification system, as has been used in the establishment of the Australian Height Datum by the Division of National Mapping. When more than a simple linear traverse has to be adjusted, a least squares adjustment shall be made using program LEVELONE or a program giving identical results.

8.6 ISLANDS

8.6.1 If the levels on islands closely adjacent to the Australian mainland are observed to standard third order accuracy, and are referred to mean sea level at a satisfactory tide gauge, they are deemed to be part of the Australian Height Datum.

8.7 LEGENDS ON MAPS

8.7.1 Whenever heights on maps are based on the adjustment of May 1971, the phrase "Australian Height Datum" is to be quoted in the legend though there is no objection to its being followed by "Mean sea level 1966-68" if desired. On the Australian continent the map user will then know that he is using AHD heights. In Tasmania, whenever heights are based on the adjustment of October 1983, the term "Australian Height Datum (Tasmania)" should be used, followed by "Mean sea level 1972" if desired.

8.7.2 On coastal sheets which contain the phrase "Australian Height Datum", heights on islands may be above local mean sea level rather than on the same mathematical model as the mainland.

8.7.3 Some care may be needed on bathymetric and hydrographic maps around the coast. If they contain heights which have been instrumentally connected to adjustments on the Australian Height Datum, they should refer to the Australian Height Datum. If heights refer only to a local tide gauge, and have not been connected to the Australian Height Datum, the legend should say "Heights based on mean sea level at .... tide gauge."

8.7.4 The datum for soundings on hydrographic charts is a low tide level, and its relationship to mean sea level and the Australian Height Datum may vary from chart to chart, and in some cases even within the limits of one chart. Datums used for soundings are referred to local bench marks, which may not yet have been connected to the AHD. Details of bench marks and levels used are given on all large scale charts.

9.

GEOID-SPHEROID SEPARATION IN AUSTRALIA

9.1 INTRODUCTION

The geoid can be defined as the equipotential surface of the earth's gravitational and rotational field which is approximated by sea level. The force and direction of gravity vary from one region to another due to variations in the composition and density of the material comprising the earth's crust. Where a deficiency of mass exists, the surface of the geoid will dip below the surface of a spheroid which best fits the geoid overall. Conversely, where a surplus in mass exists, the surface of the geoid will rise above this mean spheroid. The deviation between the geoid and spheroid is termed *geoid-spheroid separation* and is denoted by the letter N, being positive when the geoid is above the spheroid. The concept of geoid-spheroid separation is shown diagrammatically in Figures 2.2 and 2.3.

Discussion of the deflection of the vertical and its relationship with geoid-spheroid separation is beyond the scope of this manual. For further information, see Division of National Mapping Technical Report 13, *The Geoid in Australia—1971.*

9.2 THE SITUATION IN 1966

In 1966, when all geodetic surveys in Australia and what was then New Guinea were recomputed and adjusted on the Australian Geodetic Datum, the spheroidal height of Johnston Geodetic Station was adopted to be 571·2 metres which equalled the geoidal height as determined by trigonometrical levelling. In other words, due to the almost complete lack of observed N values in Australia and New Guinea at that time, it was assumed that N was equal to zero not only at Johnston but also at all other geodetic stations in the 1966 national adjustment. Consequently, all distances computed in this adjustment were geoidal, and not spheroidal, distances.

9.3 GEOIDAL PROFILING BETWEEN 1966 AND 1971

Between 1966 and 1971, a considerable number of geoidal profiles were observed throughout Australia and also in New Guinea using the astro-geodetic levelling technique. The data obtained from these observations were combined with gravimetrically computed data to produce a total of approximately 3000 values of N. Early in 1971 these values were contoured to produce a geoid map which would enable the first truly spheroidal adjustment of the national geodetic network. Although a choice of N = − 8 metres at Johnston Geodetic Station would have balanced positive and negative values of N over the whole continent, it was decided to adopt a value of N = − 6 metres for this initial geoid map in order to minimise the overall effect of a variation in scale introduced by changes in spheroidal height (N + H). Early versions of the Geodetic Model of Australia were computed in terms of N = − 6 metres at Johnston Geodetic Station.

9.4 EFFECT OF NATIONAL LEVELLING ADJUSTMENT IN MAY, 1971

The national levelling adjustment of May 1971 determined the height of Johnston Geodetic Station to be 566·3 metres above the geoid. However, the Australian Geodetic Datum as adopted by the National Mapping Council and subsequently proclaimed in the *Commonwealth Gazette* No. 84 of 6 October 1966 specified a spheroidal height of 571·2 metres. Therefore, the legal geoid-spheroid separation value to be used in Australia is 571·2 (h) - 566·3 (H) = +4·9 metres (N) at Johnston Geodetic Station.

9.5 GEOID MAP, 1971, AND COMPUTATION OF SPHEROIDAL DISTANCES

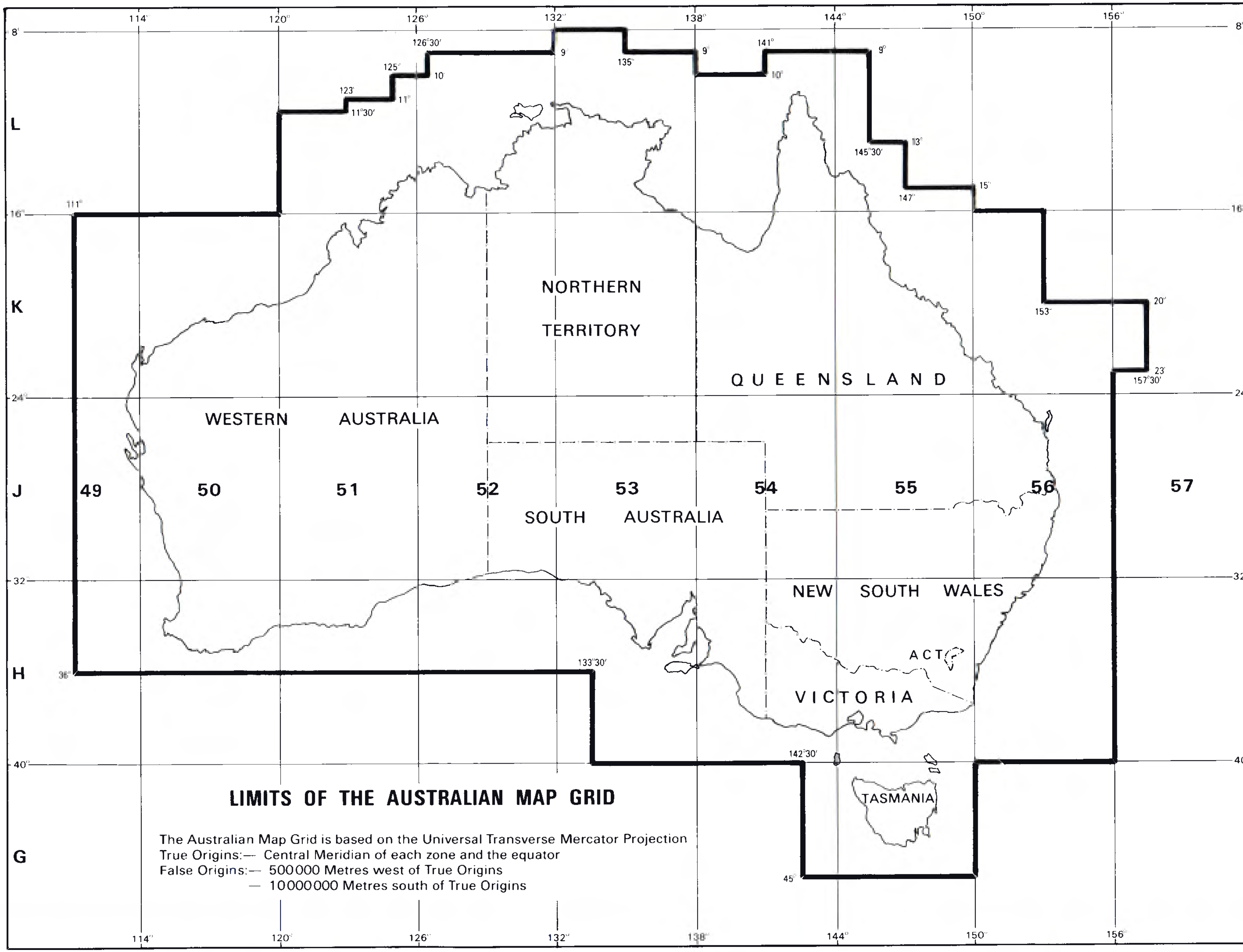
9.5.1 The geoid map of Australia attached at Annex E, which is contoured in terms of N = + 4·9 metres at Johnston Geodetic Station, should be used for converting geoidal, or sea level, distances to equivalent distances on the Australian National Spheroid. For example, it is required to reduce geoidal distances in the Mount Isa region of Queensland to equivalent Australian National Spheroid distances. As the geoid map at Annex E shows N = + 14·4 metres for this region, geoidal distances must be reduced by 2·3 parts per million in order to convert them to equivalent distances on the Australian National Spheroid. A variation of the spheroidal height of the order of ±6 metres will respectively increase or decrease the spheroidal distance by 1 part per million. Care must be exercised when dealing with long lines located in areas of relatively steep geoidal slope and in applying the correct sign to the conversion factor.

9.5.2 In Australia, geoid-spheroid separation reaches a maximum value of approximately 23 metres in terms of the legal N value of + 4·9 metres at Johnston Geodetic Station. Therefore, an error in excess of 3·6 parts per million will be introduced into computations on the Australian National Spheroid unless the geoidal contours are taken into consideration. The correction required to reduce a geoidal, or sea level, distance to a distance on the Australian National Spheroid is given by

$$s - s' = -\frac{Ns'}{R_\alpha}$$

where s = spheroidal distance
s′ = geoidal, or sea level, distance
N = mean N value for the line (with sign)
$R\alpha$ = radius of curvature of the spheroid in azimuth α

9.5.3 Some of the current generation of geodetic adjustment programs, including GANET, contain inbuilt geoid models which automatically reduce observations to the surface of the spheroid. The program used by the University of New South Wales for the GMA82 adjustment contained such a model. Due to the different technique employed, the geoid-spheroid separation values computed by these programs will differ slightly from those interpolated from the diagram at Annex E. However, these differences are not significant when reducing observations to the spheroid. For further information, the reader is referred to paragraph *2.4* of Division of National Mapping Technical Report 33, *Geodetic Model of Australia 1982.*

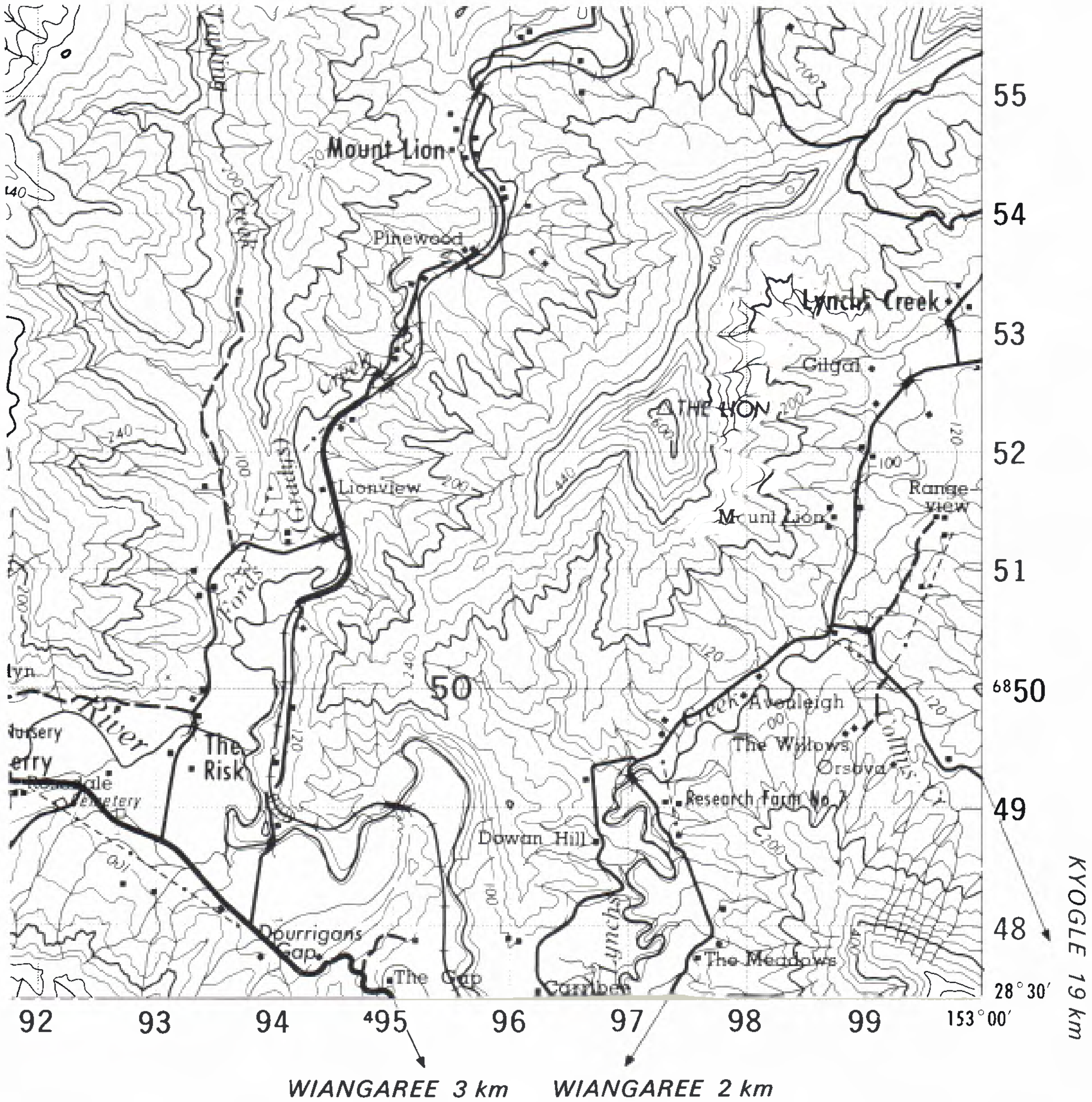


LIMITS OF THE AUSTRALIAN MAP GRID

The Australian Map Grid is based on the Universal Transverse Mercator Projection
True Origins:— Central Meridian of each zone and the equator
False Origins:— 500000 Metres west of True Origins
— 10000000 Metres south of True Origins

Grid lines covering a map

ENLARGED EXTRACT FROM THE MAP SHEET 9441
NATIONAL TOPOGRAPHIC MAP SERIES MOUNT LINDESAY



Grid Reference Boxes

Four figure references on a map at 1:250 000

UNIVERSAL GRID REFERENCE
BEFORE GIVING A GRID REFERENCE, CIVILIAN USERS SHOULD STATE THE NUMBER AND NAME OF THIS MAP:
SH56-2 WARWICK

GRID ZONE DESIGNATION: 56J	TO GIVE A STANDARD REFERENCE ON THIS SHEET TO NEAREST 100 METRES		
100 000 METRE SQUARE IDENTIFICATION	SAMPLE POINT: △ *THE LION*		
LQ MQ (690); LP MP; LN MN (680); 40	1 Read letters identifying 100 000 metre square in which the point lies:	MP	
	2 Locate first VERTICAL grid line to LEFT of point and read LARGE figures labelling the line either in the top or bottom margin, or on the line itself:	9	
	3 Estimate tenths from grid line to point:	7	
	4 Locate first HORIZONTAL grid line BELOW point and read LARGE figures labelling the line in either the left or right margin, or on the line itself:	5	
IGNORE the SMALLER figures of any grid number; these are for finding the full co ordinates. Use ONLY the LARGER figures of the grid number; example: 685000	5 Estimate tenths from grid line to point:	2	
	SAMPLE REFERENCE:	MP 9752	
	If reporting beyond 18° in any direction, prefix Grid Zone Designation, as:	56JMP 9752	

Blue numbered grid lines are 10 000 metre intervals of the Australian Map Grid, Zone 56 grid values are shown in full only at the south west corner of the map "When estimating the tenths part of a grid reference, the figure should always be rounded down".

Six figure references on map at 1:100 000 and larger

UNIVERSAL GRID REFERENCE
BEFORE GIVING A GRID REFERENCE, CIVILIAN USERS SHOULD STATE THE NUMBER AND NAME OF THIS MAP:
9441 MOUNT LINDESAY

GRID ZONE DESIGNATION: 56J	TO GIVE A STANDARD REFERENCE ON THIS SHEET TO NEAREST 100 METRES		
100 000 METRE SQUARE IDENTIFICATION	SAMPLE POINT: △ *THE LION*		
MQ (6900); MP; 500	1 Read letters identifying 100 000 metre square in which the point lies:	MP	
	2 Locate first VERTICAL grid line to LEFT of point and read LARGE figures labelling the line either in the top or bottom margin, or on the line itself:	97	
	3 Estimate tenths from grid line to point:	3	
	4 Locate first HORIZONTAL grid line BELOW point and read LARGE figures labelling the line in either the left or right margin, or on the line itself:	52	
IGNORE the SMALLER figures of any grid number; these are for finding the full co ordinates. Use ONLY the LARGER figures of the grid number; example: 452000	5 Estimate tenths from grid line to point:	3	
	SAMPLE REFERENCE:	MP973523	
	If reporting beyond 18° in any direction, prefix Grid Zone Designation, as:	56JMP 973523	

Black numbered grid lines are 1000 metre intervals of the Australian Map Grid, Zone 56 grid values are shown in full only at the south west corner of the map "When estimating the tenths part of a grid reference, the figure should always be rounded down".

WORLD GEODETIC SYSTEM

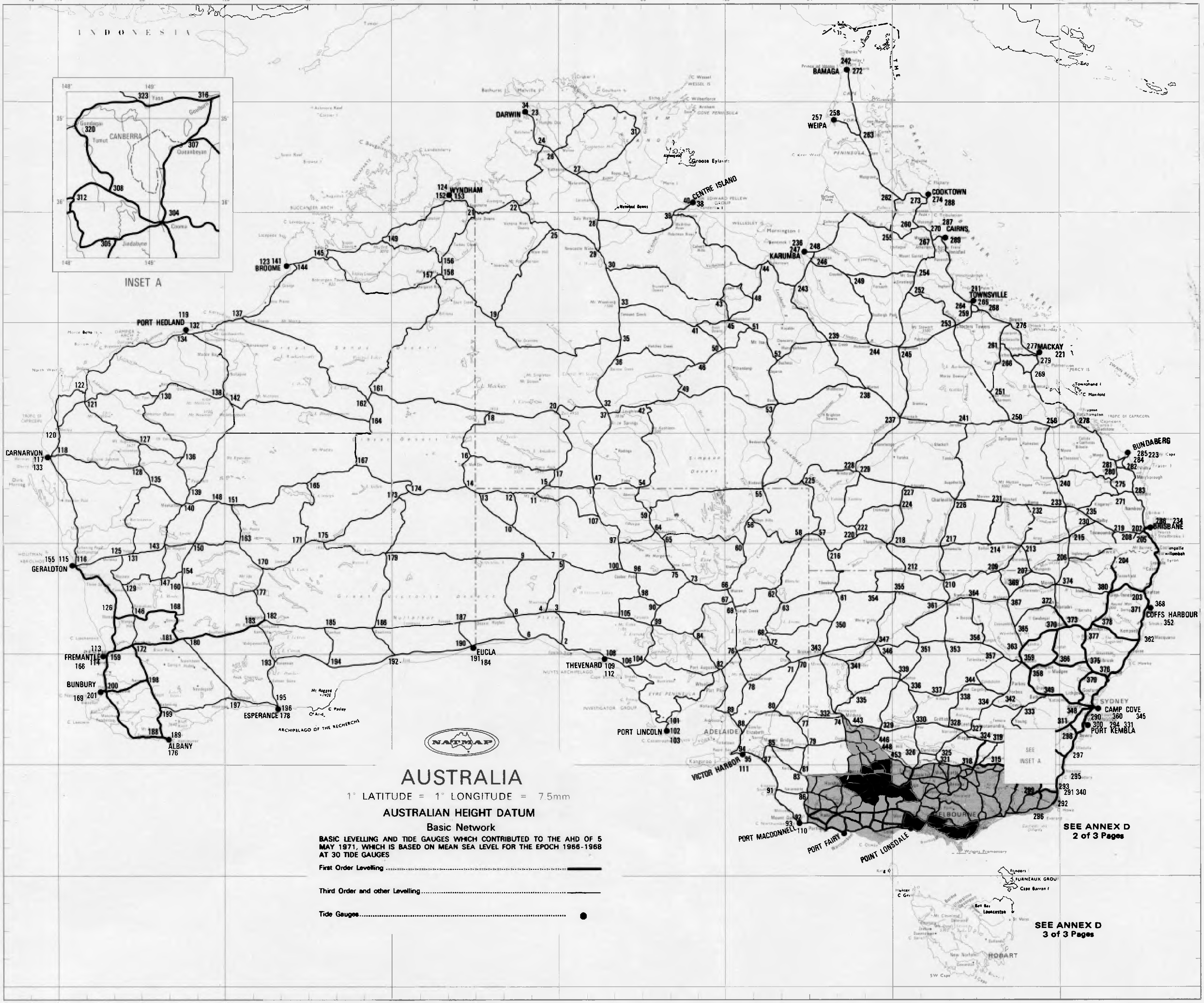
To convert World Geodetic System 1972 to Australian Geodetic Datum 1966 coordinates on which this map is based:

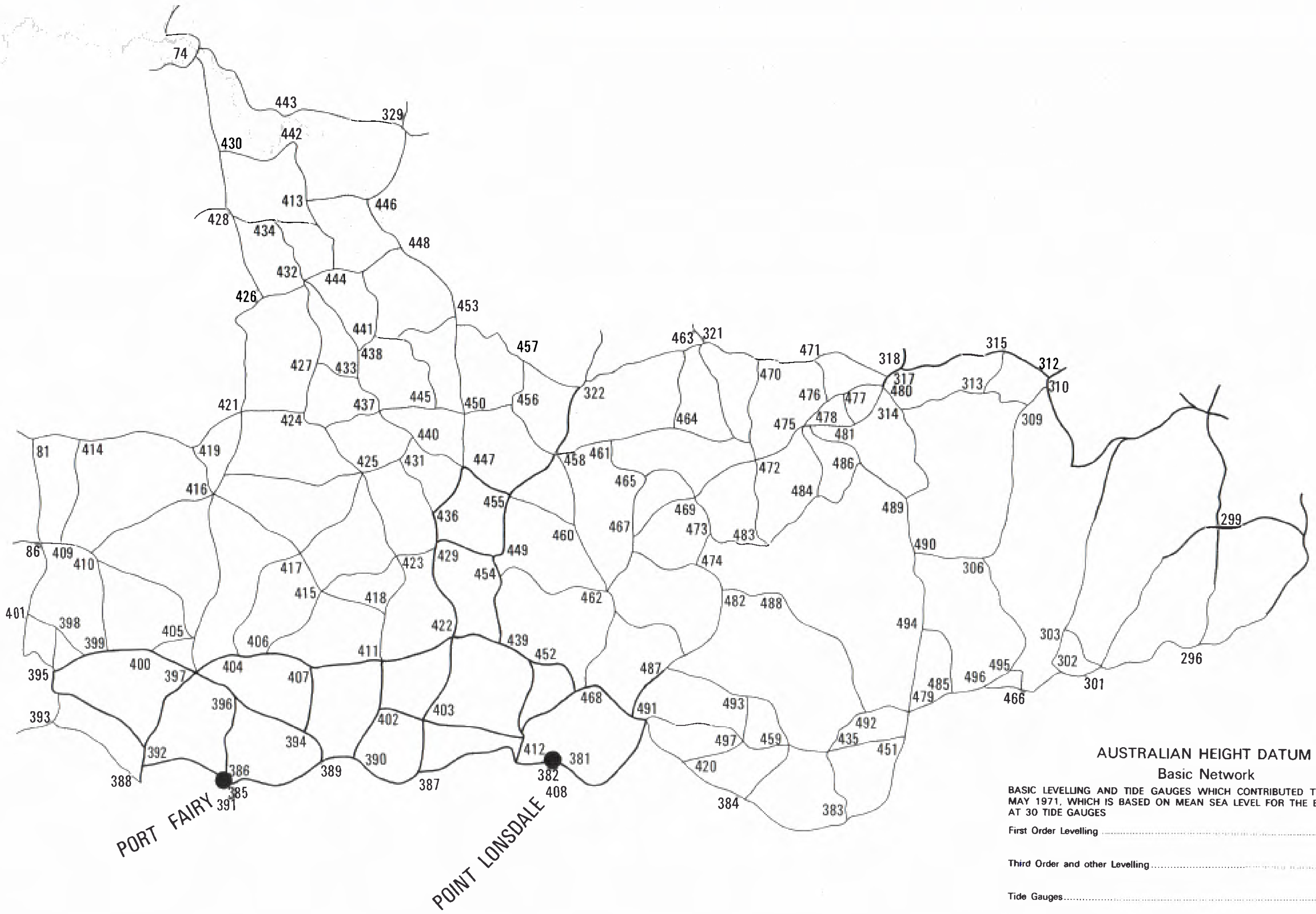
Increase the numerical value of latitudes by 5.6", equivalent to 172 metres
Decrease the numerical value of longitudes by 3.3", equivalent to 90 metres
To obtain heights above mean sea level, decrease satellite heights by 40 metres

HUNDRED THOUSAND METRE SQUARE IDENTIFICATION — AUSTRALIAN MAP GRID AREA

100 000 METRE SQUARE IDENTIFICATION

1. The identification letters of 100 kilometre squares on the AMG and UTM Grid are determined according to the conventions outlined below. A map showing the identification letters on the AMG is attached as part of this annex. Departures from the sequential lettering system are made at spheroid junctions where complex and changing situations are likely to occur. They will not be explained in this manual.
2. Where the AMG limits fall in the sea, the departures for spheroid junction conditions have not been applied, as no ambiguity is expected to occur. A concession has been made in the northern part of Zone 54, where the letters WD, WE and WF have been retained to accord with existing mapping. When a geocentric datum is adopted, these letters will revert to the normal conventions, and will therefore be changed to WP, WQ and WR respectively.
3. The 100 kilometre squares are identified by a combination of two alphabetical letters: first, the column letter and second, the row letter.
4. The 100 kilometre columns, including partial columns along zone junctions, are lettered alphabetically, A through Z (with I and O omitted), north and south of the Equator, starting at the 180° meridian and proceeding easterly. It will be seen that at the Equator there are eight whole or part 100 kilometre squares in each zone. Thus the lettering sequence repeats every 18°.
5. The 100 kilometre row lettering is based on a 20 letter alphabetical sequence, read from south to north:
   (a) The row letters in each odd numbered 6° projection zone are read in an A through V (I and O omitted) sequence commencing at the Equator. The sequence is repeated every 2 000 kilometres. This sequence continues south of the equator, again reading from south to north.
   (b) In each even numbered 6° projection zone, the same lettering sequence is advanced five letters to F, continued sequentially through V and followed by A through V. This advancement lengthens the distance between 100 kilometre squares of the same identification.


AUSTRALIA
1° LATITUDE = 1° LONGITUDE = 7.5mm
AUSTRALIAN HEIGHT DATUM
Basic Network
BASIC LEVELLING AND TIDE GAUGES WHICH CONTRIBUTED TO THE AHD OF 5 MAY 1971, WHICH IS BASED ON MEAN SEA LEVEL FOR THE EPOCH 1966-1968 AT 30 TIDE GAUGES
First Order Levelling
Third Order and other Levelling
Tide Gauges
NATMAP
INSET A
CANBERRA
SEE INSET A
SEE ANNEX D 2 of 3 Pages
SEE ANNEX D 3 of 3 Pages
INDONESIA
DARWIN
WYNDHAM
BROOME
PORT HEDLAND
CARNARVON
GERALDTON
FREMANTLE
BUNBURY
ALBANY
ESPERANCE
EUCLA
THEVENARD
PORT LINCOLN
ADELAIDE
VICTOR HARBOR
PORT MACDONNELL
PORT FAIRY
POINT LONSDALE
MELBOURNE
SYDNEY
CAMP COVE
PORT KEMBLA
COFFS HARBOUR
BRISBANE
BUNDABERG
MACKAY
TOWNSVILLE
CAIRNS
COOKTOWN
WEIPA
BAMAGA
KARUMBA
CENTRE ISLAND
HOBART



AUSTRALIAN HEIGHT DATUM

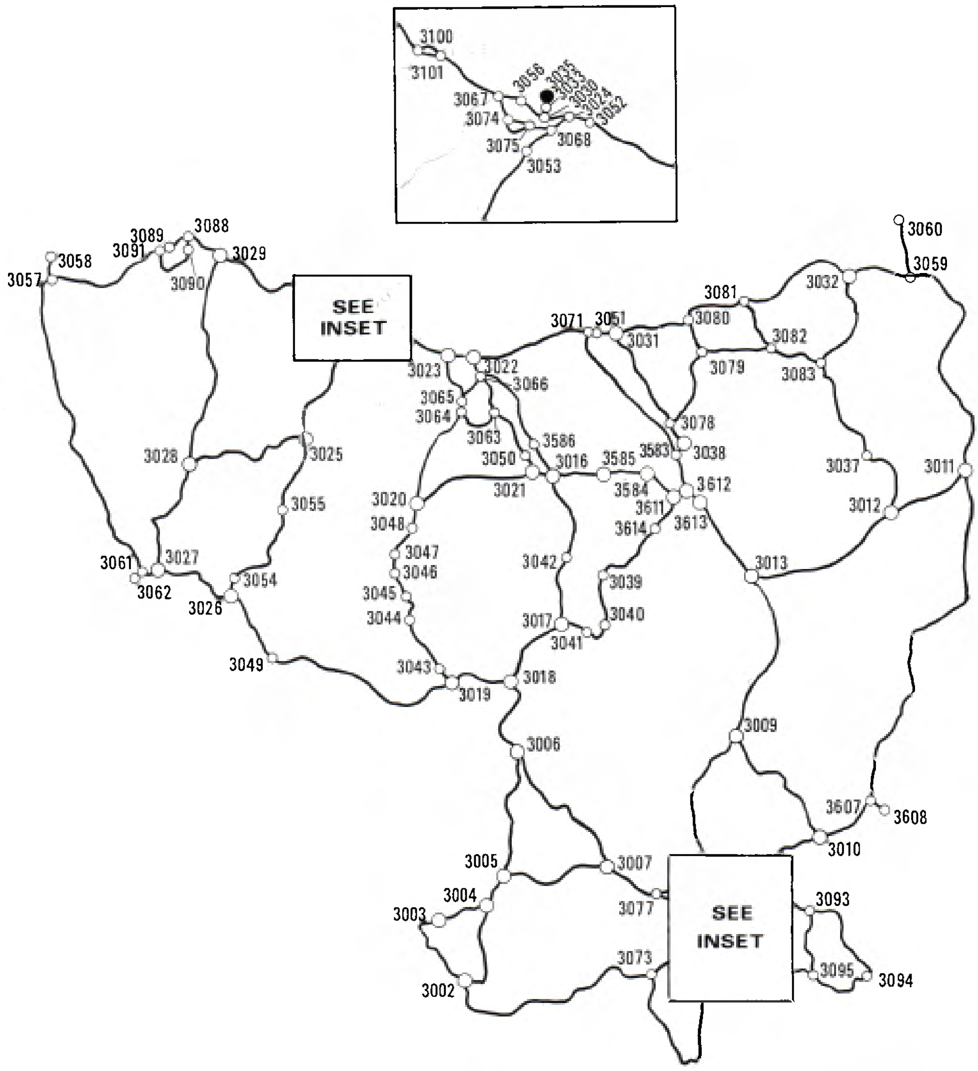
Basic Network

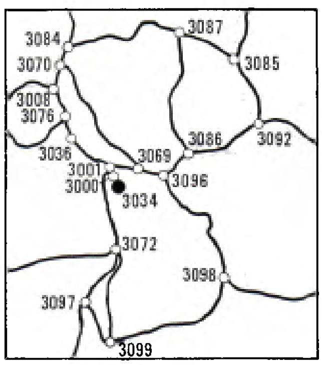
BASIC LEVELLING AND TIDE GAUGES WHICH CONTRIBUTED TO THE AHD OF 5 MAY 1971, WHICH IS BASED ON MEAN SEA LEVEL FOR THE EPOCH 1966-1968 AT 30 TIDE GAUGES

First Order Levelling ....

Third Order and other Levelling ....

Tide Gauges ....



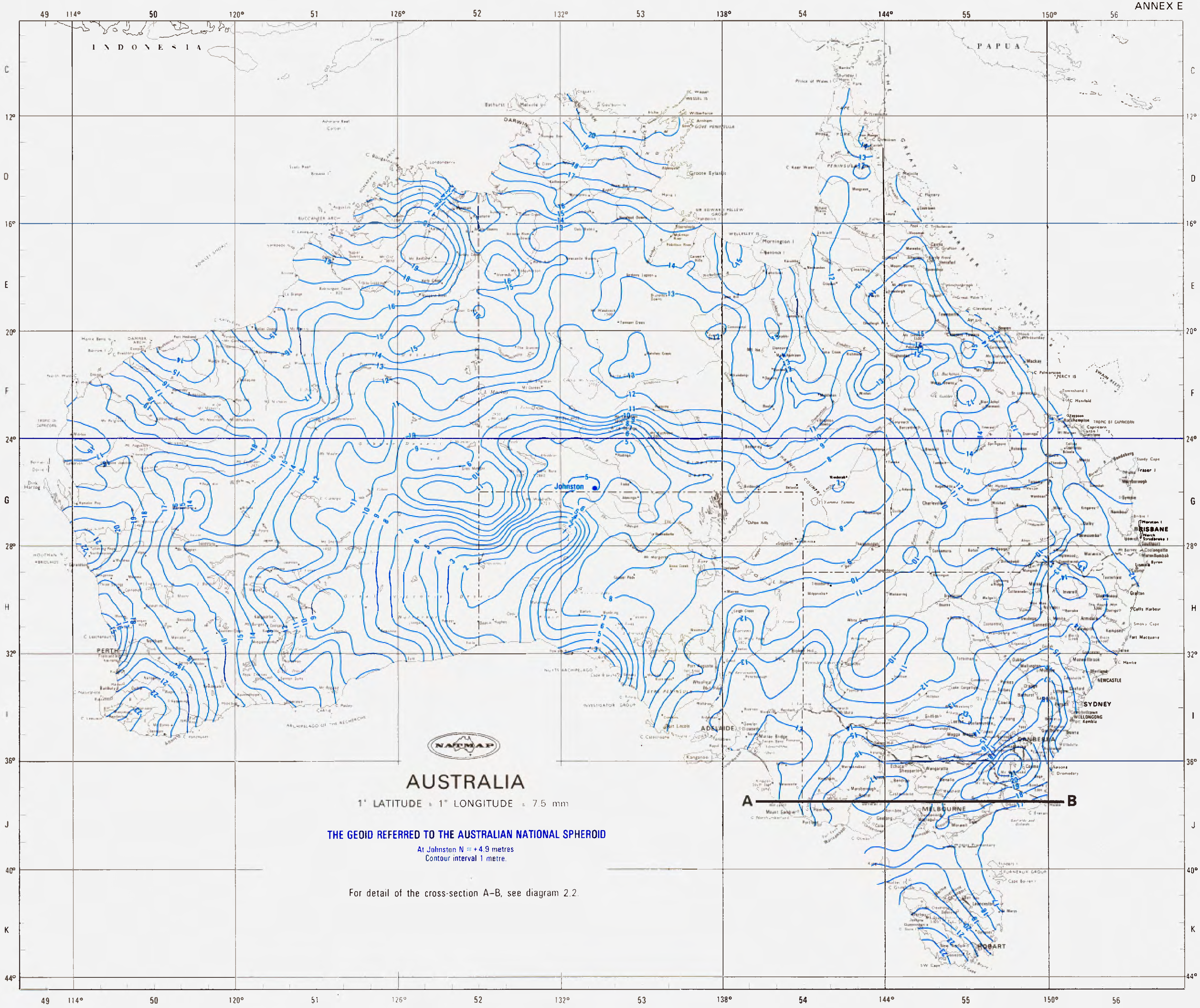


AUSTRALIAN HEIGHT DATUM
(Tasmania)—Basic Network

BASIC LEVELLING AND TIDE GAUGES WHICH CONTRIBUTED TO THE AHD (TASMANIA) OF 17 OCTOBER 1983, WHICH IS BASED ON MEAN SEA LEVEL FOR THE EPOCH 1972 FOR THE TIDE GAUGES AT HOBART AND BURNIE

Third Order Levelling .... ———

Tide Gauges .... ●


AUSTRALIA
1° LATITUDE = 1° LONGITUDE = 7.5 mm
THE GEOID REFERRED TO THE AUSTRALIAN NATIONAL SPHEROID
At Johnston N = +4.9 metres
Contour interval 1 metre.
For detail of the cross-section A–B, see diagram 2.2.
NATMAP
INDONESIA
PAPUA
Johnston
PERTH
ADELAIDE
MELBOURNE
CANBERRA
SYDNEY
BRISBANE
HOBART
NEWCASTLE
A
B

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Hydrographer, RAN
Department of Defence - Navy
Russell Offices
CANBERRA ACT 2600

Commonwealth Surveyor-General
Australian Survey Office
Department of Local Government and Administrative Services
PO Box 2
BELCONNEN ACT 2616

LENGTH OF ONE SECOND OF ARC OF LATITUDE AND LONGITUDE

Latitude

One second of arc at the equator = 30·715 metres
One second of arc at the pole = 31·026 metres

Longitude

One second of arc at the equator = 30·922 metres
One second of arc at 25° latitude = 28·042 metres
One second of arc at 45° latitude = 21·902 metres
One second of arc at 54° latitude = 18·216 metres
One second of arc at 68° latitude = 11·617 metres
One second of arc at 80° latitude = 5·387 metres

DEFINITION OF THE METRE AND THE VELOCITY OF LIGHT

The definition of the metre in terms of the orange-red line of Krypton-86 was adopted in 1960. This definition has now been replaced by an improved definition, brought about in part by the development of very stable lasers and techniques for measuring the frequency of a light wave, which has led to a very precise knowledge of the velocity of light, c_0, in vacuum:

$$c_0 = 299\,792\,458 \text{ metres per second}$$

In October, 1983, by international agreement, a new method of defining the metre was adopted:

> "The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

(Resolution 1, 17th Conférence Général des Poids et Mesures, 20th October 1983, Paris.)

This definition preserves the actual size of the unit of length, but is more precise than the previous definition.
In 1957, the XIIth General Assembly of the International Scientific Radio Union recommended that the following value for the velocity of light be adopted for general usage:

$$c_0 = 299\,792{\cdot}5 \pm 0{\cdot}4 \text{ km s}^{-1}$$

This value, which is used for the scale definition of all electronic distance measuring equipment (EDM), was later accepted by the International Union for Geodesy and Geophysics (IUGG).

In 1975, the XVIth General Assembly of the IUGG, recommended that the following revised value for c_0 be adopted:

$$c_0 = 299\,792\,458 \pm 1{\cdot}2 \text{ m s}^{-1}$$

The standard deviation of c_0 in this definition amounts to 0·004 ppm, being mainly due to uncertainty in the definition of the metre. In EDM, c_0 can therefore be considered to be an error free value, as the accuracy of EDM is rarely better than 1 ppm. The change between the 1957 and 1975 value of c_0 is 42 $m.s^{-1}$ or 0·14 ppm; it may usually be ignored but not for precise, long range EDM with an accuracy greater than 0·1 ppm.

INTERNATIONAL NAUTICAL MILE

One international nautical mile = 1 852 metres
(Adopted by the International Hydrographic Conference, 1929)

CONVERSION RATIOS

The conversion ratios laid down in the *Weights and Measures (National Standards) Regulations, 1961,* for use in Australia are as follows:

1 yard = 0·9144 international metre
1 foot = 1/3 yard

whence 1 foot = 0·3048 international metre exactly

PRECISION OF COMPUTERS

Numbers of bits used	*Number of significant digits*
16	3
32	6
48	11
64	16
80	21

RIGOROUS NUMERICAL VALUES FOR THE TEST LINES COMPUTED FROM REDFEARN'S AND ROBBINS'S FORMULAE

TEST LINE: BUNINYONG–FLINDERS PEAK

	Zone 54		*Zone 55*	
Station:	*Buninyong*	*Flinders Peak*	*Buninyong*	*Flinders Peak*
Latitude	- 37°39′15·″557 1	- 37°57′09·″128 8		
Longitude	+ 143 55 30· 633 0	+ 144 25 24· 786 6		
Easting	758 053· 090	800 817· 407	228 742·077	273 629·436
Northing	5 828 496· 974	5 793 905· 650	5 828 074·208	5 796 305·236
Azimuth	127°10′27·″08	306°52′07·″34	127°10′27·″08	306°52′07·″34
Grid Convergence	+ 1 47 16· 67	+ 2 06 25· 53	- 1 52 46· 36	- 1 35 06· 76
Grid Bearing	**128 57 43· 75**	**308 58 32· 87**	**125 17 40· 72**	**305 17 00· 58**
Arc-to-Chord	+ 23· 94	- 25· 18	- 20· 67	+ 19· 47
Plane Bearing	**128 58 07· 69**	**308 58 07· 69**	**125 17 20· 05**	**305 17 20· 05**
Point Scale Factor	1·000 420 30	1·000 714 68	1·000 506 41	1·000 231 18
Meridian Distance	- 4 169 144· 533	- 4 202 244· 285		
Rho, ρ	6 359 277· 924	6 359 600· 684		
Nu, ν	6 386 142· 439	6 386 250· 478		
Spheroidal Distance	54 972·161		54 972·161	
Line Scale Factor	1·000 563 72		1·000 364 62	
Grid Distance and Plane Distance	**55 003·150**		**54 992·205**	
Meridian Convergence	18′19·″74		18′19·″74	
Line Curvature	49·″12		40·″14	

TEST LINE: "M"–"X"

	Zone 58		*Zone 59*	
Station:	*"M"*	*"X"*	*"M"*	*"X"*
Latitude	- 29°03′23·″153 0	- 28°52′35·″171 0		
Longitude	+ 167 57 06· 632 0	+ 168 29 57· 152 3		
Easting	787 420· 487	841 341· 166	203 196· 647	256 092· 465
Northing	6 782 165· 201	6 800 667· 210	6 781 926· 377	6 803 133· 270
Azimuth	69°37′50·″00	249°21′55·″67	69°37′50·″00	249°21′55·″67
Grid Convergence	+ 1 26 04· 59	+ 1 41 29· 34	- 1 28 53· 35	- 1 12 29· 83
Grid Bearing	**71 03 54· 59**	**251 03 25· 01**	**68 08 56· 65**	**248 09 25· 84**
Arc-to-Chord	- 14· 38	+ 15· 22	+ 15· 07	- 14· 12
Plane Bearing	**71 03 40· 21**	**251 03 40· 23**	**68 09 11· 72**	**248 09 11· 72**
Point Scale Factor	1·000 619 55	1·001 038 12	1·000 687 22	1·000 334 21
Meridian Distance	- 3 215 523· 135	- 3 195 573· 356		
Rho, ρ	6 350 473· 178	6 350 303· 150		
Nu, ν	6 383 176· 604	6 383 119· 636		
Spheroidal Distance	56 959·832		56 959·832	
Line Scale Factor	1·000 822 848		1·000 504 957	
Grid Distance and Plane Distance	**57 006·701**		**56 988·594**	
Meridian Convergence	15′54·″33		15′54·″33	
Line Curvature	29·″58		29·″19	

All computed terms have been rounded as follows:

Latitude and Longitude	0·″000 1
Angles, Azimuths and Bearings	0·″01
Distances	0·001 metres

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INDEX